Design of the Resonance Frequency of Smart Helmholtz Resonator Using Response Surface Method and Optimization Analysis

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Abstract
In this paper we carried out the simulation for controlling the resonance frequency of a Helmholtz resonator, which is used for the reduction of noise. The resonance frequency is designed to the desired value by PID control of the acoustical boundary condition of the Helmholtz resonator. The resonator like this is called Smart Helmholtz Resonator. At first the gains of PID controller are changed to several values and it is confirmed that the resonance frequency is varied. In order to realize the desired resonance frequency , the gains of PID controller are determined by the response surface method and the optimization analysis. The accuracy of several cases for which the number of resonance frequencies is changed is compared and the error analysis is carried out. It is shown that this method is effective for designing the resonance frequency of Smart Helmholtz Resonator.

A Helmholtz resonator has been known from ancient times and is frequently used when an acoustical system is designed since it can reduce sound pressure level of a specified frequency. Moreover, in order to reduce the exhaust noise of cars it is sometimes used due to the characteristics that it can reduce the noise of frequency different from the resonance frequency. For a Helmholtz resonator the model using one degree of freedom or the analysis using an equivalent acoustical circuit was carried out, but the studies which apply the control theory to a Helmholtz resonator have not been reported except the reference [2].

In this study the control theory is applied to a Helmholtz resonator and the optimization of the resonance frequency of a “Smart Helmholtz Resonator” , which has a variable resonance frequency, is tried by the simulation. Namely, the resonance frequency is controlled and the desired resonance frequency is realized by PID control of acoustical boundary condition of a resonator. In the past the authors have tried to obtain the gains of PID control by using Kitamori’s model matching method, but the desired result cannot be obtained[3],[4]. Therefore, in this study the gains of PID control are determined by a response surface method and optimization analysis. Derived gains are used for obtaining the resonance frequency, it is compared with the desired frequency by simulation and the validation of our method is checked.

In Fig.1 the schematic diagram of a Smart Helmholtz Resonator is shown.

Fig.1 Schematic Diagram of a Smart Helmholtz Resonator

When the boundary condition is represented by the transfer function G(s) as shown in Fig.1, the transfer function of a Smart Helmholtz Resonator is obtained as follows.

\[
\frac{u_a}{P_{in}} = \frac{1}{I_a} \frac{s + G(s)C_a}{s^2 + \frac{G(s)}{C_a}s + \frac{1}{C_aI_a}}
\]  

(1)

Here \( u_a \) is the acoustical velocity and \( P_{in} \) is acoustic pressure. If this boundary condition is realized by PID controller, G(s) is represented as follows.

\[
G(s) = K_p + \frac{K_i}{s} + K_ds
\]  

(2)

When this is substituted to the equation (1), the transfer function of the PID controlled Smart Helmholtz Resonator is obtained as follows.
\[ u_a = \frac{1}{P_{ca}} \left[ \frac{(K_D + C_a)S^2 + K_p S + K_I}{(K_D + C_a)S^3 + K_p S^2 + (K_I + 1/I_a)S} \right] \]

Here,
\[ I_a = \rho_0 l_e \quad C_a = \frac{V}{\rho_0 c_0^2 S} \]

where \( \rho_0 \) is the density of the medium, \( S \) is the area of the cross section of the neck of the cavity, \( V \) is the volume of the cavity, \( l_e \) is the length of the neck, and \( c_0 \) is the speed of sound.

Response surface is constructed under the following condition. At (a) the proportional gain is not considered since the proportional gain does not change the resonance frequency as shown from the preliminary result, at (b) three gains are considered, at (c) the number of response is increased for investigating the effect of the number of the response. For the obtained gains the resonance frequency is obtained again from the Bode diagram and the accuracy is compared. As a result the accuracy is better for lower frequencies as shown in Table 1. In order to improve accuracy it is considered that the frequency points in response surface is increased and more calculation points are used to construct response surface.

Or, the following method can be considered that the frequency range is divided to several blocks and for each block the response surface is constructed and the gains are obtained. It is possible to perform error analysis for obtaining accuracy. How to construct PID controller for acoustical system physically is also the future problem. Moreover, performing the experiment is the remained problem to validate this method using response surface method.

### Table 1 One Result for Desired Resonance Frequency and the Obtained Resonance Frequency

<table>
<thead>
<tr>
<th>Desired Freq.(Hz)</th>
<th>Obtained Freq.(Hz)</th>
<th>Error(Hz)</th>
</tr>
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<tr>
<td>15.92</td>
<td>13.72</td>
<td>-2.21</td>
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<tr>
<td>23.89</td>
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<td>2.15</td>
</tr>
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<td>31.85</td>
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<td>47.77</td>
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<td>55.73</td>
<td>54.82</td>
<td>-0.92</td>
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<tr>
<td>63.69</td>
<td>60.78</td>
<td>-2.91</td>
</tr>
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### References