Continuous spin-up and dynamo in a precessing sphere

Yuki Koike,[1] **Masaki Shimizu,**[1] **Shigeo Kida,**[2] **Genta Kawahara,**[1] **Susumu Goto** [1]
[1]Osaka University, Graduate School of Engineering Science, Toyonaka 560-8531 Japan
[2]Doshisha University, Kyotanabe, 610-0394 Japan

1 INTRODUCTION

Flows in a precessing cavity such as a sphere, spheroid and cylinder have been studied extensively since the seminal laboratory experiment by Malkus (1968) and the analytical theory by Busse (1968). However, despite of the simple motion of cavity, flows in a precessing cavity are too complex to be fully understood on the basis of analytical theories or experiments. Hence, numerical simulations have been playing an important role to investigate characteristics of flows in a precessing cavity.

Recently, the numerical simulation by Kida and Shimizu [1] discovered a turbulent ring in a precessing sphere, along which strong vorticity as well as magnetic flux are generated. Recall that quite a few geophysicists are interested in the dynamo action in the precessing cavity, since the spin axis of the Earth is precessing slowly. In the present study, we investigate flow structures at lower Reynolds numbers and reveal the mechanism to create the ring.

2 GOVERNING EQUATIONS

We consider the MHD dynamo driven by incompressive flows in a precessing sphere with the magnitude of the spin and precession angular velocities being constant in time and two axes being orthogonal. The evolution equations in the sphere for the fluid velocity $\boldsymbol{u}(\boldsymbol{r},t)$ and the magnetic flux density $\boldsymbol{b}(\boldsymbol{r},t)$ may be written in the precession frame (x,y,z) which is rotating with the precession angular velocity $\boldsymbol{\Omega}_{\rm p} = \Omega_{\rm p}\hat{\boldsymbol{z}}$ as

$$\nabla\cdot\boldsymbol{u} = 0, \quad \nabla\cdot\boldsymbol{b} = 0,$$

$$\frac{\partial\boldsymbol{u}}{\partial t} = \boldsymbol{u}\times(\nabla\times\boldsymbol{u}) - 2\Gamma\hat{\boldsymbol{z}}\times\boldsymbol{u} - \nabla P - \boldsymbol{b}\times(\nabla\times\boldsymbol{b}) + \frac{1}{Re}\nabla^2\boldsymbol{u},$$

$$\frac{\partial\boldsymbol{b}}{\partial t} = \nabla\times(\boldsymbol{u}\times\boldsymbol{b}) + \frac{1}{Re_m}\nabla^2\boldsymbol{b},$$

where P is the modified pressure including the centrifugal force potential, $\Gamma = \Omega_{\rm p}/\Omega_{\rm s}$ the Poincare number, $Re = a^2\Omega_{\rm s}/\nu$ the Reynolds number, $Re_m = a^2\Omega_{\rm s}\mu\sigma$ the Magnetic Reynolds number, $\Omega_{\rm s}$ the spin angular velocity taken in the x direction, a the sphere radius, ν the kinematic viscosity, μ the magnetic permeability and σ the electrical conductivity of fluid. The length has been normalized by a, the time by $1/\Omega_{\rm s}$, and the magnetic flux density by $\sqrt{\rho\mu}a\Omega_{\rm s}$. The outside of the sphere is assumed to be vacuum, where the magnetic flux density $b^{(o)}$ obeys $\nabla\cdot\boldsymbol{b}^{(o)} = 0$ and $\nabla\times\boldsymbol{b}^{(o)} = 0$. These equations are supplemented by (on $r = 1$),

$$\boldsymbol{u} = \hat{\boldsymbol{x}}\times\boldsymbol{r}, \quad \boldsymbol{b} = \boldsymbol{b}^{(o)} \quad (\text{on} \ \ r=1)$$

which are the boundary conditions derived from the assumptions that the flow is non-slip on the boundary and that the magnetic permeability of the fluid is equal to that of vacuum. We also assume that $b^{(o)}$ is zero at infinity. Here note that the control parameters of this system are the Poincare number Γ, the Reynolds number Re and the Magnetic Reynolds number Re_m.

3 NUMERICAL METHOD

We solve the above set of equations numerically by spectral method. The velocity and magnetic fields are expressed by the toroidal and poroidal scalar functions U, W, B and J as $u = \nabla\times(\nabla\times\boldsymbol{r}U) + \nabla\times\boldsymbol{r}W$ and $b = \nabla\times(\nabla\times\boldsymbol{r}B) + \nabla\times\boldsymbol{r}J$. These scalar functions are expanded by the Zernike spherical polynomials and spherical harmonics; for example,

$$U(\boldsymbol{r},t) = \sum_{m=-M}^{M}\sum_{l=|m|}^{L}\sum_{\substack{n=|l|\\ n+l=\text{even}}}^{N} U_{nlm}(t)\Phi_n^l(r)Y_l^m(\phi,\theta)$$

where $\Phi_n^l(r)$ is the Zernike polynomial and $Y_l^m(\phi,\theta)$ is the spherical harmonics, (M,L,N) is the number of truncation modes, and (r,θ,ϕ) is the spherical polar coordinate with θ being the polar angle from the z-axis and ϕ the azimuthal angle from the x-axis. The evolution equations for the expansion coefficients U_{nlm} and the corresponding ones for W, B and J are derived and integrated numerically by the Adams-Bashforth and Crank-Nicolson schemes. We use $(M,L,N) = (63,63,126)$

4 RESULTS

Flow states are shown by symbols in figure 1 for the simulated combinations of Re and Re_m. Here we fix $\Gamma = 0.1$, since it has been experimentally shown that the flow becomes unsteady at relatively low Re when the Poincare number Γ is around 0.1.[2] It is interesting to observe that in all the cases of these simulation parameters there exists a high speed stream running approximately along the equator, and that the boundary layer near the wall is swelled into the interior along the ring. This high speed ring is shown in figure 2 by plotting high vorticity regions for a laminar case at Re=1500 and a turbulent case at $Re = 10000$. These rings are inclined slightly from the spin axis, and the angle of inclination seems independent of the Reynolds number. The axially averaged velocity, $\langle \boldsymbol{u} \rangle_\phi$, is shown in the right panel of figure 3 for $Re = 1500$. Note that this velocity field seems quite similar to that of spin-up.

Although it is difficult, due to the nonlinearity, to describe analytically this velocity field, the formation of the ring structure may be understood qualitatively as follows. Because the spin axis rotates about the precessing axis, fluid inside the sphere continuously tries to approach the solid-body rotation which is suitable to the boundary condition at the instance, but which is never established. It takes the duration of the order of $1/\Omega_s$ to form the boundary layer and to start the spin-up. During this period, the ring with high angular velocity inclines at an angle of the order $\Omega_p/\Omega_s = \Gamma$. It becomes unstable as Re number increases and the turbulent regions with high-activity of vorticity and magnetic flux are created along it.

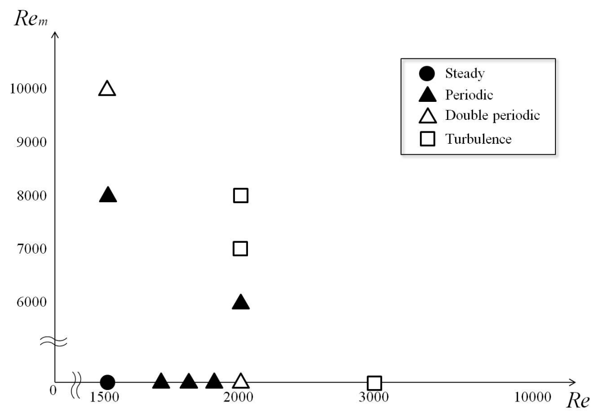

Fig. 1: Flow states at $\Gamma = 0.1$.

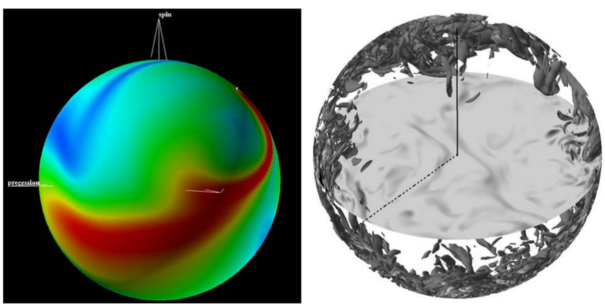

Fig. 2: Ring structure. The region of large vorticity is shown by iso-surface. *Left*: Re = 1500. *Right*: Re = 10000.

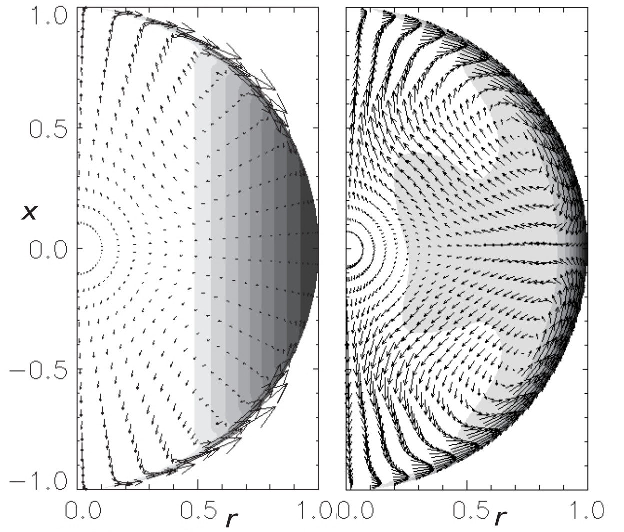

Fig. 3: Spin-up. *Left*: Velocity field during spin-up from rest at $Re \sim 5000$. *Right*: Axial averaged velocity at $Re = 1500$ and $Re_m = 8000$. In-plane components are represented by allows and the perpendicular component by gray scale.

5 CONCLUSION

Solving the MHD equations by the spectral method, we have conducted highly precise numerical simulations of fluid motions and MHD dynamos in a precessing sphere. In our simulations for the Poincare number fixed at 0.1, a ring-like high-speed stream is always observed irrespective of the Reynolds numbers. At higher Reynolds numbers, turbulent vortical structures and magnetic flux are produced along the ring (see the right panel of figure 2), whereas this ring is also observed in laminar flows at lower Reynolds numbers (see the left panel of figure 2). These observations suggest that such a laminar solution is likely to be embedded in the turbulence, and may well be key ingredients to understand the mechanism to sustain the strong turbulence and the MHD dynamo. In addition, it has been shown, in the present study, that the ring-like structure is created by a continuous spin-up process.

[References]

[1] Kida, S. and Shimizu, S., *13th European Turbulence Conference*, 268, (2011)

[2] Goto, S. et al., *Phys. Fluids*, **19**,061705 (2007)