Periodicity of Huge Earthquakes by Numerical Simulation

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1 Introduction

The massive earthquake on March 11, 2011 in Japan brought us down enormous damage. Though it is well-known that earthquakes happen almost periodically, it has not been explained exactly. The well-known empirical rule is that small scale ones often occur, however large ones rarely occur. Gutenberg and Richter formulated it as exponential rule, that is the number of occurrences decrease exponentially with the scale of earthquakes[1]. It is called Gutenberg-Richter rule and described as follows,

$$\log N(m > M) = a - bM \tag{1}$$

where M is the magnitude and $N(m > M)$ is the number of occurrences of earthquakes which is larger than M. The coefficient b is called b value and is an important parameter which shows the property of earthquakes. It is estimated between 0.7 to 1.1 in real earthquakes all over the world.

There are many researches of the relation between the magnitude of an earthquake and its released energy. In most of them, they suppose the formula $\log E_s = \alpha + \beta M_s$. Gutenberg and Richter estimated as follows,

$$\log E = 11.8 + 1.5M \tag{2}$$

so the power-law in $N(m > M)$ and E as follows

$$N(m > M) \propto E^{-\frac{2}{3}b}. \tag{3}$$

The main aim of this paper is to show the periodicity of the occurrence of earthquakes in numerical simulation. The most famous model of the earthquake is the stick-slip model[2]. Using that model, we can obtain size, spacial and temporal distribution of the magnitude of earthquakes. In most researches, they only confirm Gutenberg-Richter rule, and researches of the periodicity are rare. Thus we carry out numerical simulation using stick-slip model and confirm Gutenberg-Richter rule and the periodicity of earthquakes.

2 Model and Simulation

2.1 The Stick-Slip Model

The stick-slip model expresses one fault using several blocks and springs between them. As shown in Fig.1, they have a fixed ceiling plate and an unidirectionally moving floor plate with some friction, and some blocks contact the latter. Each block connects each other with springs in four directions, and also connects the ceiling plate in a similar way. If we make a modeling of the fault around the boundary of the continental plate and the ocean plate, the floor and blocks stand for the ocean and continental plates respectively. The subduction movement of the ocean plate is expressed by the movement of the floor.

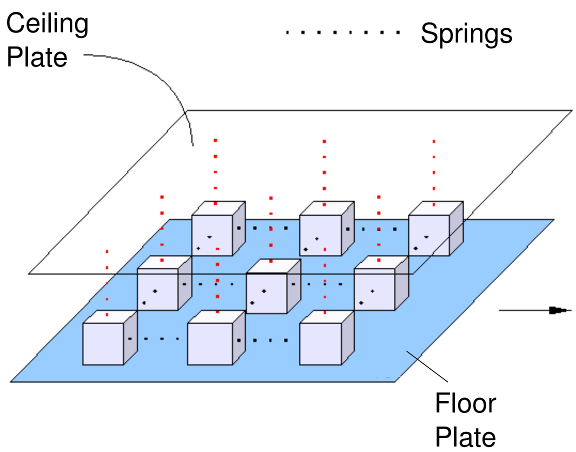

Fig. 1. The schematic diagram of stick-slip model.

As the floor moves, the elastic force of springs connected to the ceiling and other blocks becomes larger than the static friction, then some block starts to slip. The movement of that block causes stretch or shrink of other springs and increase of their elastic force. Then some surrounding blocks also start to slip in a similar way. The propagation of these slips occur successively until no other blocks slip. We consider all these processes as one earthquakes and all released spring energy means the energy of that earthquake.

2.2 Simulation

In simulation first we set placements of all blocks 0, and fix the static friction of them as $F_N = 5.0 \times 10^5$. We carry out simulation as follows.

1. Set static frictions for 100×100 blocks.
2. Set spring constants of springs between the ceiling plate and blocks and between blocks.
3. For each block, calculate the sum of elastic force of surrounding springs, and ingenerate the slip if it is larger than the static friction.
   (a) For slipping blocks, calculate the displacement of the cell after the slip.
   (b) Check whether all other blocks would slip again.
   (c) Count the number of new slipped blocks, and repeat (a) and (b) until it becomes 0.
4. Record the sum of released energy.
5. Increase the displacement of all blocks by 1.
6. Repeat from 3 to 5.

We consider the procedures from 3 to 5 as 1 step and carry out simulation until 5×10^5 steps. Here spring constants are given by random number according to normal

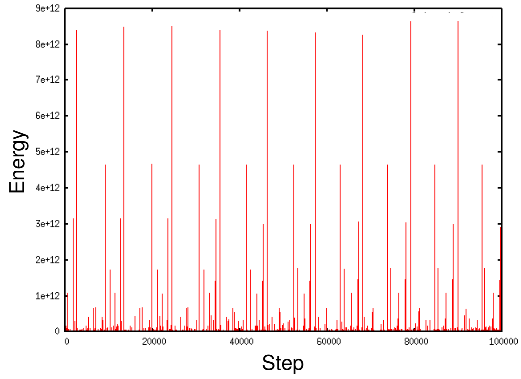

Fig. 2. The change of released energy.

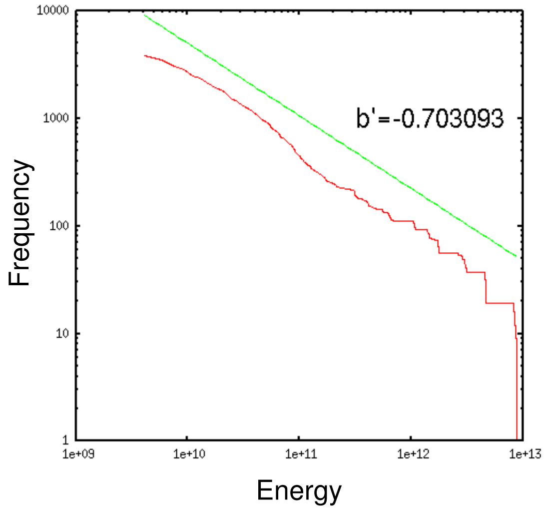

Fig. 3. The relation between energy and frequency.

distribution with the mean k and the variance σ_k^2, and we vary k and σ_k and measure the change of the periodicity and b value.

3 Results and Discussion

3.1 Verification of Gutenberg-Richter Rule

The numerical results of released energy with $k = 10$ and $\sigma_k = 0.5$ are shown in Fig.2. We can easily find earthquakes occur almost periodically, however those scales are slightly different. From this data, we can obtain the relation between energy and the number of occurrences of earthquakes as shown in Fig.3. We estimated the power exponent $b' = -0.703$ and thus from Eq.(3) $b = 1.0546$.

3.2 Periodicity and b value

We observed the periodicity and b value for several spring constants with $k = 5, 10, 20, 30, 40, 50, 60, 70, 80, 90$ and $\sigma_k = \frac{k}{20}$. Results for the periodicity are shown in Fig.4. Simulation data are well fitted by $f(x) = \frac{a}{x}, a = 1.127 \times 10^5$, thus periods are inversely proportional to spring constants. On the other hand, the relation between b value and spring constants is almost logarithmic(Fig.5).

3.3 Discussion

In stick-slip phenomena, if we think of only one block, the slip distance is proportional to the displacement, and the released energy is proportional to the square of the displacement. Therefore we can easily speculate that the larger spring constant becomes, the more frequently small earthquakes will occur, and the smaller it becomes, the less large slips occur.

In our 2-D stick-slip model, springs between blocks affect the condition of the occurrence of slip. If spring constant is strong, the propagation of slips is easy to occur. Therefore since some complicated elements affect it, the periodicity and b value are unpredictable by spring constants.

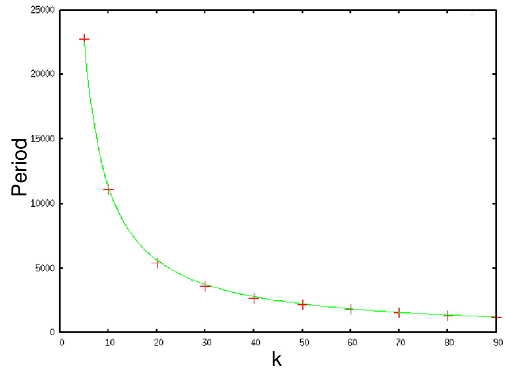

Fig. 4. The relation between periodicity and k.

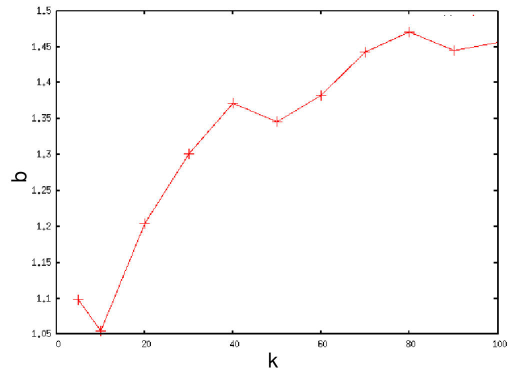

Fig. 5. The relation between b and k.

4 Summary

In summary we carry out numerical simulations of earthquake using 2-D stick-slip model. we obtained that if the spring constant is high, the period and b value of earthquake become short and high respectively. In real earthquakes the relation between the periodicity and b value is unknown though, it is known that there exist specific b value and the period of large earthquakes in each place. Since in our simulation we focus attention only on the change of spring constants showing the hardness of plates, it cannot transpose to an actual earthquake related to complicated factors as it is. However it might be contributory to the analysis and prediction of earthquakes near future.

References

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[2] R. Burridge, and L. Knopoff, Model and theoretical seismisity, Bull. Seism. Soc. Am., 57, pp.341–371, 1967.