Coherent Vorticity and Current Density Simulation of Magnetohydrodynamic Turbulence

K. Yoshimatsu[1], N. Okamoto[2,3], Y. Kawahara[1], K. Schneider[4], and M. Farge[5]

[1]Department of Computational Science and Engineering, Nagoya University, Nagoya, 464-8603, Japan
[2]Center for Computational Science, Graduate School of Engineering, Nagoya University, Nagoya, 464-8603, Japan
[3]JST, CREST, 5, Sanbancho, Chiyoda, Tokyo 102-0075, Japan
[4]M2P2–CNRS & CMI, Aix-Marseille Université, 39 rue Frédéric Joliot-Curie, 13453 Marseille Cedex 13, France
[5]LMD–IPSL–CNRS, Ecole Normale Supérieure, 24 rue Lhomond, 75231 Paris Cedex 05, France

1 Introduction

Turbulence, which is ubiquitous in our daily life, has a large number of degrees of freedom, a wide range of dynamically active scales and strong nonlinearity. A key feature of turbulence is strong intermittency, which is attributed to coherent structures, e.g. vortex tubes and sheets. Observations show its self-organization into structures, even at large Reynolds number superimposed to a background flow. This motivates us to split turbulent flows into two contributions: i.e., a coherent flow and a random background flow. Both contributions are multiscale and exhibit no scale separation. The wavelet representation is an efficient tool to perform such a multiscale decomposition, since wavelets are well localized functions in space, scale and direction. The wavelet representation has been utilized to extract coherent vorticity from turbulence with a reduced set of degrees of freedom. To simulate hydrodynamic (HD) turbulent flow using this reduced set, the coherent vorticity simulation (CVS) method has been proposed [1, 2, 3].

CVS is a deterministic computation of HD turbulent flows based on the wavelet filtered Navier-Stokes equations, using an adaptive wavelet basis while either modeling or neglecting the influence of the incoherent background flow. At each time step the vorticity field is decomposed into two orthogonal components using an orthogonal wavelet basis: the coherent vorticity, corresponding to the coefficients whose modulus is larger than a threshold, and the remaining incoherent vorticity. The threshold value depends on the total enstrophy, which evolves in time, and on the maximal resolution, which remains constant. The value is directly related to Donoho's criterion [4] which supposes the incoherent flow to be Gaussian and decorrelated. To track the translation of the coherent vorticity and the generation of smaller scales, a safety zone is required in wavelet space [5, 6].

The CVS approach is different from the filtering approach of large eddy simulation, where only the evolution of the large-scale flow is computed while modeling the influence of small-scale motion onto the large-scale motion.

The aim of the present work is a generalization of CVS in order to compute the time evolution of coherent flow in magnetohydrodynamic (MHD) turbulence which exhibits other types of intermittent dynamics. We call this simulation method coherent vorticity and current density simulation (CVCS). CVCS is carried out for three-dimensional (3D) forced homogeneous incompressible MHD turbulence without any imposed uniform magnetic field. Homogeneous turbulence is chosen here in order to demonstrate the efficiency of CVCS in the worst possible case where structures are spread all over physical space in contrast to inhomogeneous turbulence. To assess CVCS, the results are compared with direct numerical simulation (DNS) using the same maximal resolution.

2 Methodology of CVCS

In CVCS, the vorticity and current density fields are respectively decomposed at each time step into two orthogonal components, the coherent and incoherent fields, using an orthogonal wavelet representation. Each of the coherent fields is reconstructed from the wavelet coefficients whose modulus is larger than a threshold, while their incoherent counterparts are obtained from the remaining coefficients. The two threshold values depend on the instantaneous kinetic and magnetic enstrophies as well as the maximal resolution [7]. The induced coherent velocity and magnetic fields are computed from the coherent vorticity and current density using the Biot-Savart kernel. In order to compute the flow evolution, one should retain not only the coherent wavelet coefficients but also their neighbors in wavelet space, i.e., a safety zone.

CVCS is performed for 3D forced incompressible homogeneous MHD turbulence without mean magnetic field in a 2π periodic box for a magnetic Prandtl number equal to unity and with $N = 256^3$ grid points. We also carry out DNS of the turbulence to assess the CVCS. These computations are integrated over the time period $t_f = 3.31\tau$, where τ is a large-eddy turnover time. The numerical code uses a Fourier pseudo-spectral method with a fourth-order Runge-Kutta scheme for time marching. The aliasing errors are removed by a phase-shift method. Solenoidal random forces are imposed only at large scale, i.e. in the wavenumber range $1 \leq k < 2.5$. They have the same time history in all presented computations. We use the same statistically quasi-stationary initial flow field which was obtained by a preceding DNS computation of MHD turbulence. Compactly supported Coiflet wavelets with filter width 12 are used. We here perform CVCS using pseudo-adaptive computations, as done for CVS in [5] and [6], because the motivation is to get insight into the feasibility of fully adaptive CVCS computations.

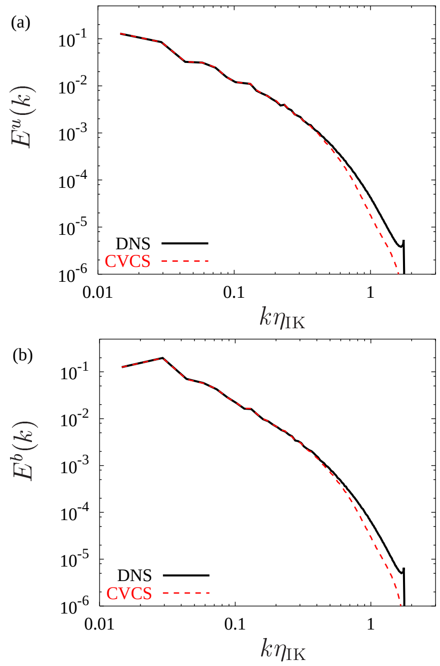

Fig. 1. (a) Kinetic and (b) magnetic energy spectra $E^u(k)$ and $E^b(k)$ at $t = t_f$. The wavenumber k is normalized by the Iroshnikov and Kraichnan microscale $\eta_{\rm IK}$ of the DNS at $t = t_f$.

3 Assessment of CVCS

CVCS retains only 13[%] of the total number of degrees of freedom after a transient decay at early times, for $t/\tau \gtrsim 0.1$ (figure omitted). However, the time evolution of the kinetic and magnetic energies in CVCS is in excellent agreement with that in DNS, during the time evolution period studied here. In Fig. 1, we find that for CVCS both of the kinetic and magnetic energy spectra, $E^u(k)$ and $E^b(k)$, agree well with those for DNS, respectively.

Figure 2 shows flow visualizations of intense vorticity regions and current density regions. Vorticity sheets and current sheets are observed. We can see that CVCS well preserves the positions of these regions of DNS at the final time t_f. This observation is in contrast to what is found in CVS of 3D homogeneous incompressible HD turbulence [6]. The CVS computations show a statistically similar picture of entangled vortex tubes as in DNS. However, the position of these intense vorticity regions in CVS is completely different from those in DNS because of the flow sensitivity. It was confirmed that impressions obtained from the visualization are the same as those at different time instants $t \leq t_f$, say, $t = t_f/2$.

Further details on the assessment of CVCS are presented in [8]. It was also found that the wavelet representation better predicts the turbulence statistics, such as the vorticity and current density probability density functions, than the Fourier representation.

4 Conclusion

We developed the CVCS method to track the time evolution of coherent vorticity and current density for 3D incompressible MHD turbulence, and examined the feasibility of CVCS for 3D forced homogeneous incompressible MHD turbulence in the absence of imposed uniform magnetic field. We found that the statistics of the reference DNS are well preserved by CVCS with safety zone, while the number of the degrees of freedom retained by CVCS is reduced by a factor eight in comparison to DNS. It was verified that information which travels further than the safety zone is not crucial to track the evolution of the nonlinear dynamics.

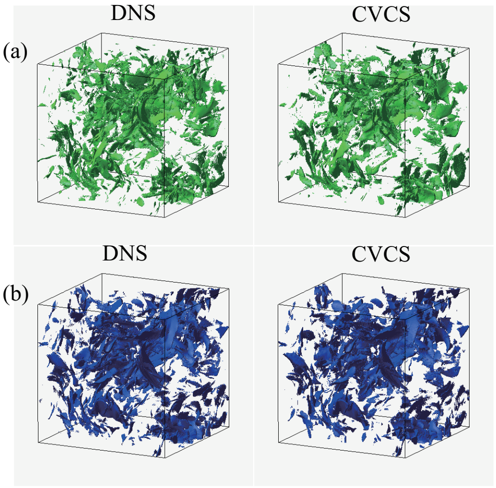

Fig. 2. Visualization of (a) the intense vorticity and (b) current density regions for DNS and CVCS at $t = t_f$. Isosurfaces of vorticity $\boldsymbol{\omega}$ and current density $\boldsymbol{j}$ are shown for $|\boldsymbol{\omega}| = \langle|\boldsymbol{\omega}|\rangle + 3\sigma_\omega$ and $|\boldsymbol{j}| = \langle|\boldsymbol{j}|\rangle + 3\sigma_j$, where $\langle\cdot\rangle$ expresses the mean value of $\cdot$. Here σ_ω and σ_j denote the standard deviations of $|\boldsymbol{\omega}|$ and $|\boldsymbol{j}|$, respectively.

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