Disentanglement of inhomogeneous and anisotropic Rayleigh–Taylor turbulence

Takeshi Matsumoto[1]

[1]Division of Physics and Astronomy, Graduate School of Science, Kyoto University, Japan

1 Introduction

The Kolmogorov's phenomenology of turbulence, which is a dimensional analysis with identification of the key quantity such as the energy dissipation rate, provides us the intuitive and quantitative understanding of the homogeneous and isotropic turbulence. However, it is very hard to extend this phenomenology to inhomogeneous and anisotropic turbulent systems. To confront this difficulty, there developed a way to "disentangle" [1] complicated knots of various inhomogeneous and anisotropic effects one-by-one. This can be achieved by expanding the statistical quantities like the longitudinal velocity structure functions with the spherical harmonics $Y_{\ell,m}(\theta,\varphi)$ as

$$\begin{aligned} S^{(p)}(\boldsymbol{x};\boldsymbol{r}) &= \left\langle \left\{ [\,\boldsymbol{u}(\boldsymbol{x}+\boldsymbol{r},\,t)-\boldsymbol{u}(\boldsymbol{x},\,t)\,]\cdot\frac{\boldsymbol{r}}{|\boldsymbol{r}|} \right\}^p \right\rangle \\ &= \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} S^{(p)}_{\ell,m}(\boldsymbol{x},r)\;Y_{\ell,m}(\theta,\varphi), \end{aligned} \tag{1}$$

which is also known as the SO(3) decomposition. We here assume that the system is in a statistically steady state. The coefficient $S^{(p)}_{\ell,m}(\boldsymbol{x},r)$ carries information of each anisotropic sector (ℓ,m) on a specified spatial point $\boldsymbol{x}$. It is believed that the coefficient has the scaling behavior

$$S^{(p)}_{\ell,m}(\boldsymbol{x},r) = C(\boldsymbol{x})\times r^{\zeta^{(p)}_{\ell,m}} \tag{2}$$

if the energy containing scale and the dissipation scale are order-of-magnitude different. This implies that the inhomogeneity does not affect the scaling exponents. Indeed, for inhomogeneous and anisotropic systems, the isotropic sector ($\ell=m=0$) is known to follow the Kolmogorov scaling, e.g., $\zeta^{(2)}_{0,0}=2/3$ [1]. With this disentangle method, one can address many questions such as how does the scaling exponent $\zeta^{(p)}_{\ell,m}$ depend on the sector (ℓ,m) ?; is there universality of these exponent values? and so on [1].

In this paper, as an inhomogeneous and anisotropic turbulent system, we numerically study so-called Rayleigh–Taylor (RT) turbulence, which is a turbulent state initiated by the Rayleigh–Taylor instability between the two miscible fluids with different densities under the effect of the gravity. If the density difference of the top and bottom fluids is very small, recent simulation studies of RT turbulence [2, 3] motivated by its Kolmogorov-type phenomenology [4] have revealed that this particular kind of RT turbulence is very close to homogeneous and isotropic turbulence in terms of the scaling laws of the velocity and density structure functions (the density behaves the same way as the passive scalar). What if the density difference is not small? We expect that there is a different scaling law or a different intermittency. We address this question with the disentanglement method.

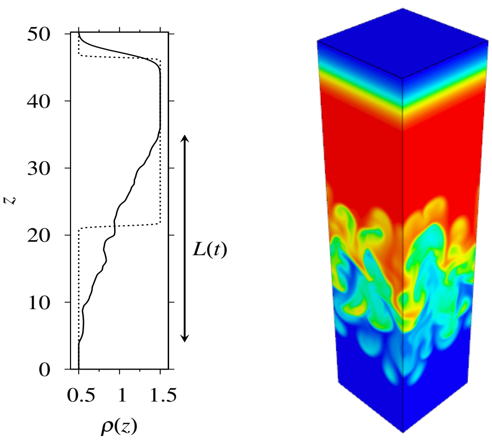

Fig. 1. Left: the horizontally averaged density profiles in the accelerated frame. The dashed line corresponds to the initial profile. The domain size is $(4\pi)^2\times 16\pi$ and the resolution is $128^2\times 512$. Right: color-coded density field showing typical plumes going up and down (Red: heavy fluid, Blue: light fluid).

2 Equations of the Rayleigh–Taylor turbulence with a moderate density difference

The density difference of the RT turbulence can be measured with the Atwood number $A=(\rho_{\text{top}}-\rho_{\text{bottom}})/(\rho_{\text{top}}+\rho_{\text{bottom}})$. In the studies mentioned in the previous section, the typical value of A was 0.10, where the Boussinesq approximation (valid for $A\ll 1$) was employed. Here we take a moderate value $A=0.50$ and use one form of the anelastic approximations proposed in [5], thereby neglecting a compressible effect. For this RT turbulence with $A=0.50$, there is numerical evidence that its mixing property is peculiar and different from that of homogeneous and isotropic barotropic turbulence [6], which is why we study this system. The equations of the RT turbulence with the anelastic approximation are

$$\partial_t\rho+\nabla(\rho\boldsymbol{u}) = 0, \tag{3}$$

$$\nabla\cdot\boldsymbol{u} = -\nabla\left(\frac{D}{\rho}\nabla\rho\right), \tag{4}$$

$$\begin{aligned} \rho[\partial_t\boldsymbol{u}+(\boldsymbol{u}\cdot\nabla)\boldsymbol{u}] &= -\nabla(p+\rho_0 gz)+A\tilde{\rho}\boldsymbol{g}+\eta\nabla^2\boldsymbol{u} \\ &\quad+\left(\frac{\eta}{3}+\zeta\right)\nabla(\nabla\cdot\boldsymbol{u}). \end{aligned} \tag{5}$$

Here $\rho, \boldsymbol{u}$ is the density and the velocity; D is the diffusion coefficient associated with the miscibility of the two fluids; ρ_0 is the reference density $\rho_0 = (\rho_{\text{top}} + \rho_{\text{bottom}})/2$; p is the pressure; $\boldsymbol{g} = (0, 0, -g)$ is the gravitational acceleration; η and ζ are viscosity coefficients. They are solved in a triply periodic domain with the standard spectral method. Mainly the result of the domain size $(4\pi)^2 \times 32\pi$ with resolution $256^2 \times 2048$ is reported here. The other parameter values are $\eta/\rho_0 = D = 10^{-3}, \zeta = 0.0$. We use the 2nd order Adams–Bashforth method for the temporal integration.

The initial density is given by a step function of the vertical coordinate z with small two-dimensional perturbations, whose profile is shown as the dashed line in Fig.1. Notice that the RT turbulence is unsteady. The simulations are stopped when the turbulent-zone length (denoted as $L(t)$ in Fig.1) reaches 60 % of the vertical domain size in order to suppress effect of the boundary condition. Repeating this process with several different initial density perturbations, we obtain an ensemble of the RT turbulence over which the average is calculated. In the calculation of the structure functions, the position $\boldsymbol{x}$ and increment $\boldsymbol{r}$ are both taken to be inside the turbulent zone, which is within $L(t)$ shown in Fig.1.

3 Disentanglement of the structure functions

In practice, the coefficients $S^p_{\ell,m}(\boldsymbol{x}, t)$ in the SO(3) decomposition Eq.1, are numerically obtained as follows. Firstly, we calculate the structure function on the Cartesian grid points of $\boldsymbol{r}$. Secondly, we interpolate the structure function data onto the Gauss-Legendre points for a suitably chosen set of $r = |\boldsymbol{r}|$'s. Lastly, we calculate the coefficients from the data on the Gauss-Legendre points with spherepack, a software library of the spherical harmonics.

We begin with Fig.2 (a) showing that the structure functions of the longitudinal velocity components without using the disentanglement method. Here, in addition to the ensemble average, the spatial average with respect to the position $\boldsymbol{x}$ inside the turbulent zone is taken. No scaling behavior is seen, which is mainly due to the low Reynolds number. However the isotropic components $S^{(p)}_{0,0}(\boldsymbol{x}, r)$ shown in Fig.2 (b) exhibit better scaling qualities, whose exponents are consistent to the Kolmogorov scaling law. This demonstrates power of the disentanglement method. Here $\boldsymbol{x}$ is taken to be the middle of the turbulent zone. The anisotropic sector $S^{(4)}_{2,m}$ of the fourth order structure function is plotted in Fig.2 (c). Again the scaling r^2 is observed, which coincides with the scaling of anisotropic barotropic turbulence. The same plots of other anisotropic sectors indicate that the scaling law of the longitudinal velocity of the RT turbulence is identical to that of the barotropic turbulence on the contrary to our expectation that a different scaling law might exist for the RT turbulence with not-small Atwood number. The inhomogeneity effect is studied by changing the position $\boldsymbol{x}$ in the turbulent zone, which shows that the scaling exponents are the same for different $\boldsymbol{x}$ as expected.

For the structure functions of the density, it appears that they obey the same isotropic and anisotropic scaling law of the passive scalar advected by the barotropic turbulence, which is again contrary to our expectation. However for the density at large scales there is an indication of a different scaling law from the Obukhov-Corrsin scaling law, which will be discussed at the conference.

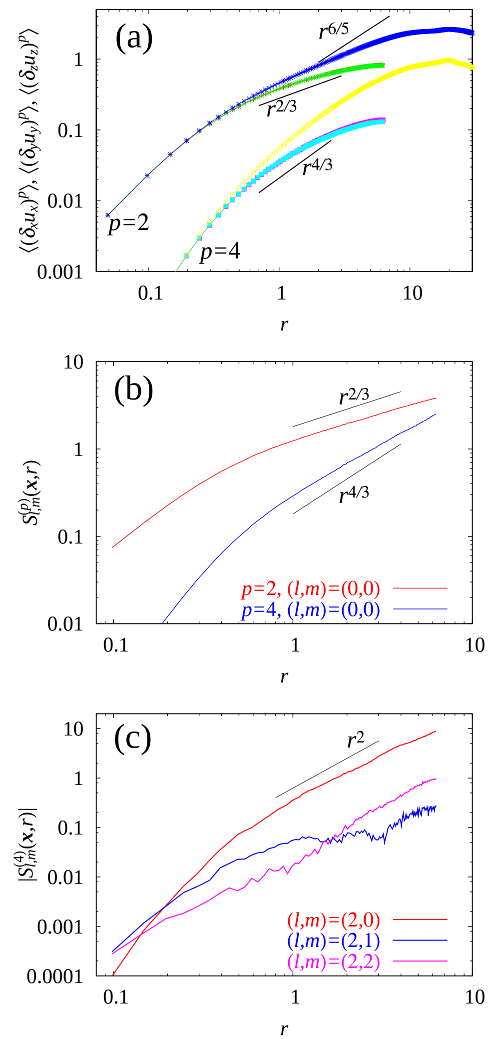

Fig. 2. (a) Second and fourth-order longitudinal velocity structure functions of the RT turbulence at late time without the disentanglement. The increments $\boldsymbol{r}$ are along with the x, y and z axes (the longer curves correspond to the z-axis case). The power laws $r^{2/3}$ and $r^{4/3}$ correspond to the Kolmogorov law and $r^{6/5}$ corresponds to the Bolgiano–Obukhov law of the stratified turbulence, which is another scaling candidate for the RT turbulence. (b) Their isotropic sectors extracted with the disentanglement method. (c) The anisotropic sectors $\ell = 2$ for the 4th-order structure function.

Acknowledgement

A part of the simulation was performed with SX8 in the Yukawa Institute of Kyoto Univerisity. This work was supported by the Grant-in-Aid for Young Scientists (B) No. 2174290 from the JSPS.

References

[1] Biferale and Procaccia, Phys. Rep., 414, 43–164, 2005.

[2] Boffetta *et al.*, Phys. Rev. E, 79, 065301(R), 2009.

[3] Matsumoto, Phys. Rev. E, 79, 055301(R), 2009.

[4] Chertkov, Phys. Rev. Lett., 91, 115001, 2003.

[5] Joseph, Eur. J. .Mech. B, 9, 565–596, 1990.

[6] Livescu and Ristorcelli, J. Fluid Mech., 591, 43–71, 2007.