The splitting of a turbulent puff in pipe flow

Masaki Shimizu,[1] **Paul Manneville,**[2] **Yohann Duguet,**[3] **Genta Kawahara**[1]

[1]Osaka University, Graduate School of Engineering Science, Toyonaka 560-8531, Japan

[2]Ecole Polytechnique, Hydrodynamics Laboratory, F-91128 Palaiseau, France

[3]LIMSI CNRS, UPR 3251, F-91403 Orsay, France

1 Introduction

In 1883, Osborne Reynolds experimentally discovered that the state of the flow in a pipe depends on a single parameter, now called Reynolds number Re, and that it changes qualitatively from laminar to turbulent at some critical Re number, $Re_c \simeq 2000$, under sufficiently large disturbance. Steady laminar flow being linearly stable for all Reynolds numbers, the transition to turbulence is caused by finite amplitude perturbations. This non-linearity makes it difficult to locate Re_c. Around Re_c, axially localized turbulent states, called "puffs", can be observed.[1] Because the length of a puff seems statistically constant at moderate Re number, it was thought to be an equilibrium state. However recent many experimental and numerical studies have revealed that a turbulent puff decays within a finite time and the statistical probability to persist can be understood as a stochastic process. Furthermore some studies have shown that the decay time does not diverge at any finite Re number. These facts make the determination of Re_c more complicated.

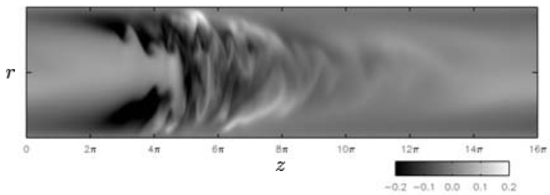

Fig. 1: Turbulent puff in pipe flow.[1] Gray scale represents perturbation velocity, $u_z - \langle u_z \rangle_{\theta z}$, in a plane through the pipe's centerline.

On the other hand, a puff can not only decay but also split. More recently, Avila et al. (2011) made a break-through in this problem. They have also shown that the splitting time can be explained by a stochastic process, and they defined Re_c as the intersection of two curves for decaying and splitting times above which turbulence is sustainable in the thermodynamic limit.[2] We consider this splitting process and show how it develops.

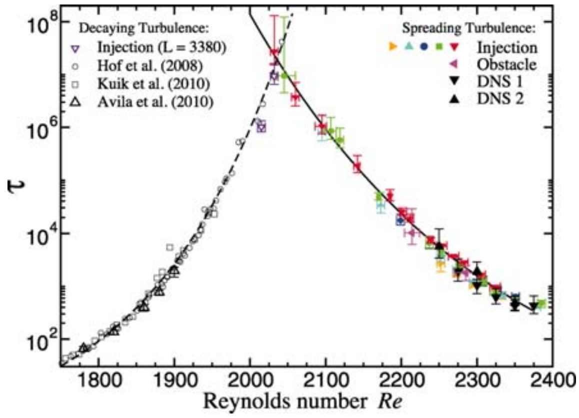

Fig. 2: Mean lifetime of a puff.[2] Dashed and solid curves represent decaying and splitting time respectively.

2 Equations

We consider the flow of an incompressible viscous fluid driven by an external force and imposed time independent mass flux U in a straight circular pipe of radius a. A cylindrical polar coordinate system (r, θ, z) is introduced to describe the velocity field $\boldsymbol{u}$ with the z axis taken on the pipe centerline. We apply no-slip conditions at the pipe wall and periodic boundary conditions in the z direction with period aL. Then the governing equations of the flow are given by the continuity equation and Navier-Stokes equations,

$$\nabla \cdot \boldsymbol{u} = 0, \tag{1}$$

$$\frac{\partial \boldsymbol{u}}{\partial t} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} = -\nabla p + \frac{1}{Re}\nabla^2 \boldsymbol{u} \tag{2}$$

together with the no-slip boundary conditions

$$\boldsymbol{u}(1, \theta, z) = 0 \tag{3}$$

and the periodic boundary conditions

$$\boldsymbol{u}(r, \theta, z + L) = \boldsymbol{u}(r, \theta, z). \tag{4}$$

Here, the Reynolds number is defined as $Re \equiv 2aU/\nu$ and, all the physical quantities have been non-dimensionalized by a for length, $2U$ for velocity and ν

being the kinematic viscosity. The reduced pressure p includes an external force which is consistent with the constant mass flux condition.

3 Numerical Scheme

The solenoidal field $\boldsymbol{u}$ can be expressed as

$$\boldsymbol{u} = \nabla \times (\psi \hat{\boldsymbol{z}}) + \nabla \times (\nabla \times (\phi \hat{\boldsymbol{z}})) . \tag{5}$$

Using a spectral method, we solve numerically the set of evolution equations for these scalar functions which are equivalent to above equations for $\boldsymbol{u}$. We approximate the scalar functions by a finite series of expansion as

$$\begin{pmatrix} \psi \\ \phi \end{pmatrix} = \sum_{k=-K}^{K} \sum_{m=-M}^{M} \sum_{\substack{n=|m| \\ n+m=\text{even}}}^{N} \begin{pmatrix} \widehat{\psi}_n^{mk} \\ \widehat{\phi}_n^{mk} \end{pmatrix} \Phi_n^m(r) \mathrm{e}^{\mathrm{i}[m\theta + (2\pi/L)kz]} \tag{6}$$

where K, M and N ($\geq M$) are positive integers and $\Phi_n^m(r)$ are Zernike circular polynomials. We take $(N, M, K) = (40, 21, 1535)$.

4 Splitting

The time variation of the streamwise velocity on the centerline is plotted in Fig. 3 that shows spreading of puffs at Re =2100,2150 and 2200. Dark regions represent disturbed flow. Each simulation starts from the same initial condition, a single puff state at $Re = 2000$. We take the streamwise period L very long, $L = 400$, to weaken the effect of the boundary conditions. At $Re = 2100$ a puff splits irregularly, whereas at $Re = 2200$ the turbulent region seems to be expand continuously while containing laminar spots at places. An example of puff splitting at $Re = 2100$ is represented in Fig. 4. A puff sometimes throws a disturbance ahead, which is associated with a low-speed streak. At lower Re, most of these disturbances dissipate (Fig.4 (a)(b)), while the others lead to the creation of a new puff ahead of the original one (Fig.4 (c)(d)). In case of a successful event, thrown-ahead disturbance spreads around in the spanwise (θ) direction after sufficient downstream motion. The frequency of these events and the success ratio increase as Re increases and the spreading gets more continuous at higher Re number.

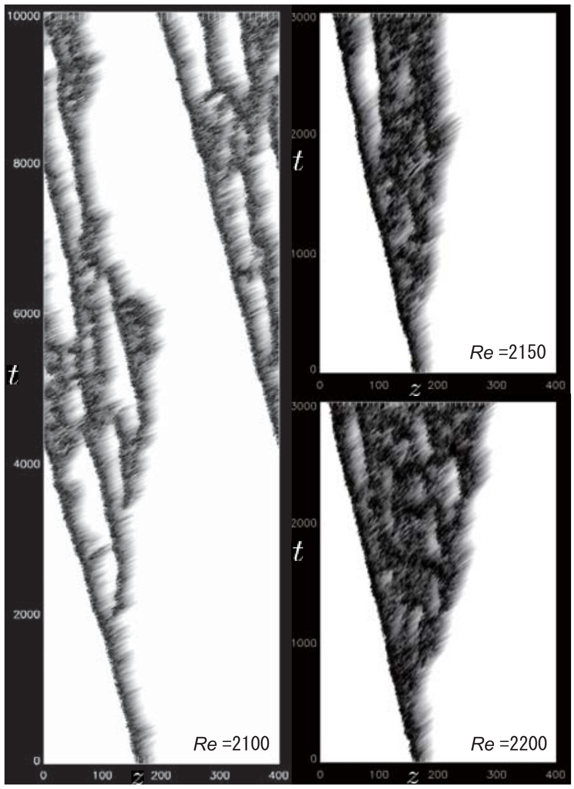

Fig. 3: Spreading of turbulence.

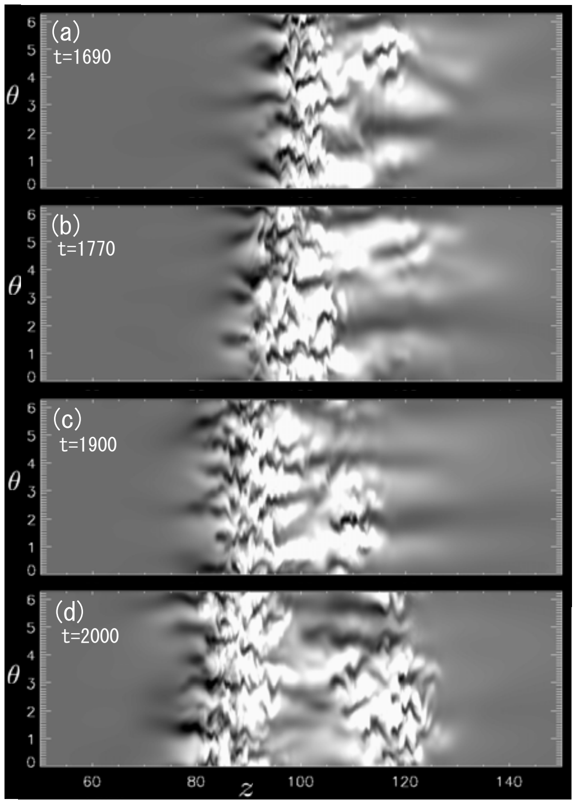

Fig. 4: Puff splitting. Streamwise velocity is plotted in the θz plane at $r = 0.8$.

[References]

[1] Shimizu, M. and Kida, S. *FDR.*, **41**, 045501 (2009)

[2] Avila, K. at al., *Science*, **333**, pp.192 (2011)