3-D Numerical Simulation of Volcanic Eruption Clouds

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1 Introduction

During explosive volcanic eruptions, a mixture of hot solid particles (volcanic ashes) and volcanic gas is released from the volcanic vent into the atmosphere. Such events are characterized by the formation of buoyant plumes and/or density currents (Fig. 1). Generally, the ejected material (i.e., volcanic ashes and gas) have an initial density of several times as large as the atmospheric density since it contains more than 90 wt.% volcanic ashes at the vent [1]. As the ejected material is mixed with ambient air, the density of the mixture drastically decreases and becomes less than the atmospheric density, because the entrained air expands by heating from the hot ashes. When the ejected material entrains sufficient air to become buoyant, a large turbulent plume ("eruption column") rises up to a height of several tens of kilometers. On the other hand, if the ejected material does not entrain sufficient air and its vertical velocity falls to zero before the eruption cloud becomes buoyant, an eruption column collapses and the heavy and hot cloud spreads radially as a density current ("pyroclastic flow"). The upper part of the pyroclastic flow entrains the ambient air and forms another type of buoyant plume ("co-ignimbrite ash cloud"). Because the impact and type of volcanic hazards are largely different between the two eruption styles (i.e., eruption column and pyroclastic flow), it has been a central subject of volcanology and hazard science to quantitatively predict the condition where an eruption column collapses to generate a pyroclastic flow.

The plume such as an eruption column and/or co-ignimbrite ash cloud exhausts its thermal energy and loses its buoyancy with the stratified atmosphere. At the neutral buoyancy level (NBL) where the plume density is equal to that of the atmosphere, the eruption cloud spreads laterally as a density current ("umbrella cloud") [1]. If the plume has much greater vertical velocity than the speed of cross-wind in the atmosphere, its trajectory is not significantly controlled by the wind and rises vertically (Fig. 1a). On the other hand, if the vertical velocity is much smaller than the cross-wind speed, the plume is largely distorted by the cross-wind and shows a bent-over trajectory (Fig. 1b); this effect can modify the efficiency of turbulent mixing, and hence, the maximum eruption column height. Because the height of eruption column is one of the few available data for estimating the eruption conditions, it is of critical importance to clarify the relationship between the eruption condition and column height under the cross-wind conditions.

The dynamics of volcanic eruption clouds are largely controlled by the efficiency of turbulent mixing between the eruption cloud and ambient air, as described above. Previous studies using steady one-dimensional (1-D) models of eruption clouds [e.g., 1] described the behaviour of turbulent mixing in a simple form based on the theoretical work of pure jet/plumes [2]. Unlike pure jet/plumes, the driving force of eruption clouds changes with height because of the change in density. The flow near the vent was negative buoyancy and is driven by the initial momentum. As the eruption column rises, the density of the cloud decreases and the flow is primarily driven by positive buoyancy. In this study, we aim to develop a numerical model of eruption cloud which can reproduce the density change of the cloud and the turbulent mixing correctly.

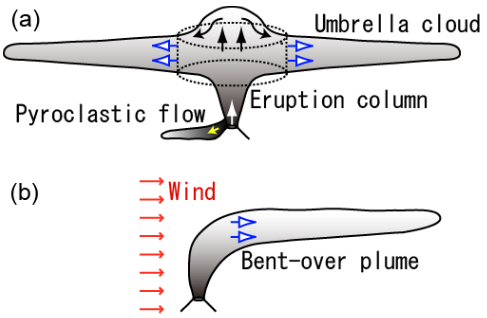

Fig. 1. Schematic illustrations of volcanic eruption clouds.

2 Model Description

The numerical model of eruption cloud is based on the work of Suzuki et al. [3]. The model is designed to describe the injection of a gas-ash mixture from a circular vent above a flat surface in a stratified atmosphere. The physical domain involves a vertical and horizontal extent of several tens of kilometers. At the ground boundary, the free-slip condition is assumed for the velocities of the ejected material and air. At the upper and other boundaries of computational domain, the fluxes of mass, momentum, and energy are assumed to be continuous. We assume steady conditions; for each run, vent radius, exit velocity, initial temperature, and initial mass fraction of gas content are fixed. In this study, we adopt a pseudo gas model; we ignore the separation of volcanic ashes from the eruption cloud. The momentum and heat exchanges between the volcanic ashes and gas are assumed to be so rapid that the velocity and temperature are the same for all phases.

2.1 Equation of State

The density of eruption clouds varies nonlinearly with the mixing ratio between the ejected material and air. We reproduce the nonlinear feature of mixture density by changing the effective gas constant and the effective specific heat of the mixture as

$$R_m = n_a R_a + n_g R_g \, , \tag{1}$$

$$C_{vm} = n_a C_{va} + n_g C_{vg} + n_s C_{vs} \, , \tag{2}$$

where R is the gas constant, n is the mass fraction, and C_v is the specific heat at constant volume. The subscripts m, a, g, and s refer to the mixture, air, volcanic gas, and solid particles (ashes), respectively. The mass fraction of air (n_a), volcanic gas (n_g), and ashes (n_s) satisfy the condition of $n_a + n_g + n_s = 1$. Using these procedures, the equation of state for the mixture of the ejected material and air can be approximated by that for an ideal gas as

$$p = \rho R_m T \,, \tag{3}$$

where p, ρ, T are the pressure, density, and temperature of the mixture. On the assumption of ideal gas, the governing equation is expressed by the Euler equation for compressible flow.

2.2 Numerical Procedure

The partial differential equations are solved numerically by the Roe scheme [4] in space, which is a general total variation diminishing (TVD) scheme for compressible flow and can simulate a generation of shock waves inside and around the high-speed jet correctly. The MUSCL method [5] is applied to interpolate the fluxes between grid points, and therefore our numerical model achieves third-order accuracy in space. These equations are solved using the time splitting method.

In order to correctly reproduce the general feature of turbulent mixing that the efficiency of entrainment is independent of the Reynolds number [6], it is essential to apply 3-D coordinates with a sufficiently high spatial resolution [3]. In this study, the calculations were performed on a 3-D domain with a non-uniform grid. The grid size was set to be sufficiently smaller than $L_0/8$ near the vent, where L_0 is the vent radius. In order to effectively simulate the turbulent mixing with high spatial resolutions both far from and close to the vent, the grid size increases at a constant rate (by a factor of 1.02) up to L_0 with the distance from the vent.

3 Results

We carried out three simulations for explosive eruptions. Table 1 lists the initial conditions and numerical conditions for the simulations. In all the runs, an initial temperature of T_0 = 1000 K and initial mass fraction of volcanic gas of n_{g0} = 0.0284.

Table 1 Input Parameters of the Simulations

Simulation	L_0 (m)	m_0 (kg s^{-1})	w_0 (m s^{-1})
Run 1	45	4.0×10^6	180
Run 2	189	1.0×10^8	115
Run 3	27	2.0×10^6	115

Our simulations have reproduced the behaviour of eruption clouds including eruption columns and/or the formation of pyroclastic flows and the unsteady and multidimensional features of eruption clouds.

Run 1 shows the typical result of eruption column (Fig. 2). The eruption cloud is ejected from the vent as a turbulent jet. As it rises and entrains ambient air, the density of the cloud becomes less than that of air and then a buoyant plume rises. The buoyant plume is high unstable as it ascends, and undergoes meandering instability that induces efficient mixing so that its radial scale gradually increases with height. After the eruption column reaches up to a height of 14 km, the eruption cloud flows down to the NBL (~10 km) and spreads laterally as an umbrella cloud.

When the vent radius is larger (189 m in Run 2), a buoyant plume and pyroclastic flow develop simultaneously (figure is not shown). Some parts of the mixture remain heavier than air, whereas the others become lighter. The heavier parts collapse to the ground and spread radially as a pyroclastic flow. The lighter parts of the mixture continue to rise as a buoyant plume and form an umbrella cloud. The upper region of the pyroclastic flow entrains air and forms co-ignimbrite ash clouds which subsequently join the buoyant plume.

Simulation of Run 3 reproduces the bent-over plume of eruption cloud (Fig. 3). Near the vent, the eruption cloud rises vertically. Above 3km, the cloud is largely distorted by the cross-wind. Then, the eruption cloud stops to rise and eventually spreads horizontally along the wind direction around the height of 5-8 km.

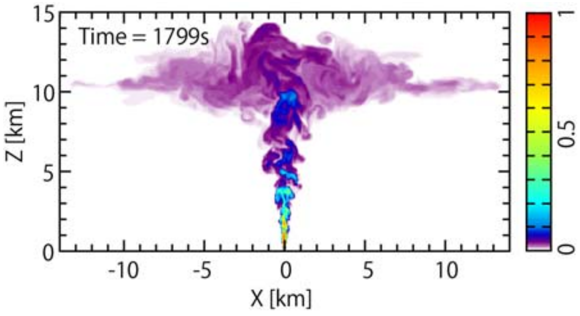

Fig.2 Numerical simulation result of eruption column and umbrella cloud in Run 1. Cross-sectional distribution of the mass fraction of the ejected material in x-z space at

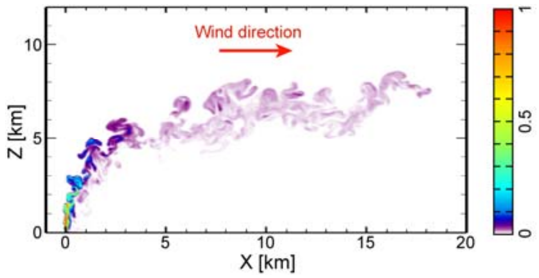

Fig.3 Numerical simulation result of bent-over eruption plume in Run 3. Cross-sectional distribution of the mass fraction of the ejected material in x-z space in 406 sec is shown.

4 Conclusion

In this study, we have developed a new numerical model of the dynamics of eruption clouds. Employing 3-D coordinates, a 3rd order accuracy scheme and a fine grid size, our model can successfully reproduce an eruption column, pyroclastic flow, umbrella cloud, and bent-over plume. These results enable us to estimate the relationship between the eruption conditions and the observable data.

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