Puff–spot transition in rectangular-duct flow

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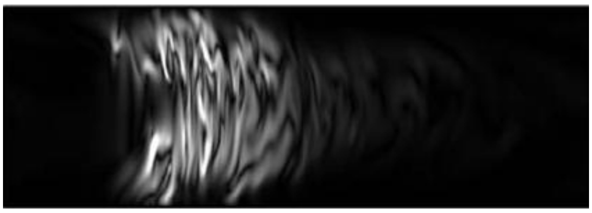
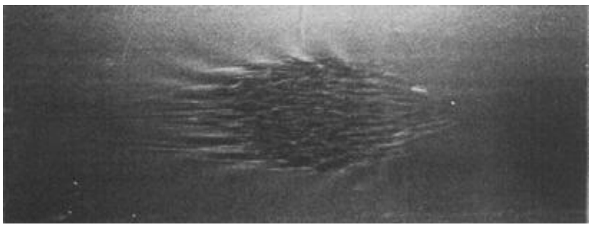
1 Introduction

Generally, the motion of fluids is smooth and laminar at low speeds, but it becomes highly disordered and turbulent as the velocity increases. The onset of turbulence in the flow through a long circular straight pipe has intrigued scientists since Reynolds' experimental investigations[1] at the end of the 19th century. Despite more than a century of research, the observed turbulent motions remain largely mysterious. The principle issue is that the flow is linearly stable[2,3], i.e. it remains laminar for all Reynolds number. However most practical pipe flows are turbulent. Hence there is a direct conflict between the theory and experiments.

In the transitional region, turbulence coexists with laminar flow, and the localized turbulent structures survive. Fig.1 shows the localized turbulent structures at the transitional Reynolds numbers. Notice that, in the pipe flow, turbulence occurs in the form of localized patches called puffs, which are embedded in the surrounding laminar flow and are localized in the streamwise direction. In the plane channel flow, on the other hand, turbulence occurs in the form of localized patches called spots, which are localized in the streamwise direction and in the spanwise direction.



(a) The turbulent puff structure in a pipe flow.



(b)The turbulent spot structure in a plane channel flow.

Fig. 1. The localized turbulent structures at the transitional Reynolds numbers.

When the aspect ratio is small, the rectangular duct flow is close to the pipe flow, so the localized turbulent structure becomes a turbulent puff. When the aspect ratio is large, the rectangular duct flow is close to the plane channel flow, so the localized turbulent structure becomes a turbulent spot. However, it is not known at what aspect ratio the transition from laminar to turbulent flow takes place. In the present study, conduct direct numerical simulations of the rectangular-duct flow to investigate the transition dependence on the aspect ratio.

We also study on the marginal Reynolds numbers in the rectangular-duct flow. For the plane channel flow, the marginal Re number is found by a numerical experiment[4]. Recently the marginal Re number in the pipe flow has been revealed by a direct numerical simulation and an experiment[5]. However, it is not known how the marginal Reynolds numbers depend on the aspect ratio in the rectangular duct.

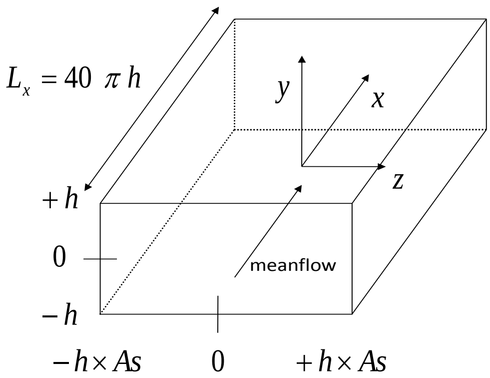


Fig. 2. Computational model of the rectangular-duct flow.

2 Numerical Methods

Fig. 2 shows the coordinate system: the x-axis is in the streamwise direction, the y-axis is in the duct-height direction and the z-axis is in the spanwise direction. We fix the grid numbers of the duct-height direction and of the streamwise direction, and change the grid numbers of the spanwise direction. We have numerically integrate the Navier-Stokes equations of an incompressible fluid by the spectral method.

We decrease the Reynolds number by 5 from an initial condition, which is a fully turbulent state at Re=1500 (here $Re=Uh/\nu$, where U is the mean velocity, h is the duct half-height and ν is the kinematic viscosity of the fluid). If the localized turbulent structure survives for the duration of 400+ after decreasing the Reynolds number, we conclude that the turbulent structure survives (here the symbol + denotes the unit normalized by the wall friction velocity and the kinematic viscosity of the fluid). If the localized turbulent structure disappears before 400+, we conclude that the flow becomes fully laminar. The marginal Re number is evaluated as the minimum Re number, in which the localized turbulent structure survives.

3 Result

Fig. 3 shows a result of visualization of vortical structures from the positive direction of the y-axis near the sidewall around the marginal Reynolds numbers. Here, to visualize the vortex structure is used positive isosurfaces of the second invariant of velocity gradient tensor Q (Q^{+}=2.0 × 10^{-3}). To distinguish turbulent puffs from turbulent spots, we introduce a new index to evaluate the strength of the turbulence in the vicinity of the sidewalls ($z/h=\pm As$, here As is the aspect ratio.);

$$\left\langle \omega_x^2 \right\rangle_{wide} = \frac{1}{2A}\left(\int_{x_1}^{x_2} \int_{-As\,h}^{As\,h} \omega_x^2 (y/h = \pm 1) \right) dz dx$$

$$\left\langle \omega_x^2 \right\rangle_{narrow} = \frac{1}{2A}\left(\int_{x_1}^{x_2} \int_{-h}^{h} \omega_x^2 (z/h = \pm As) \right) dy dx$$

In the streamwise direction we integrate these indices over the internal $x_1 \le x \le x_2$ where $\omega_x' \ge 0.5\max(\omega_x')$ is satisfied. Here ω_x' is the cross-sectional mean vorticity strength,

$$\omega_x' = \sqrt{1/A \iint_A \omega_x^2 \mathrm{d}y\mathrm{d}z}$$

Fig. 4 shows that the vortical structures near the sidewall disappear in the vicinity of As = 4.0. That is, the localized turbulent structures change from turbulent puffs to turbulent spots around As = 4.0.

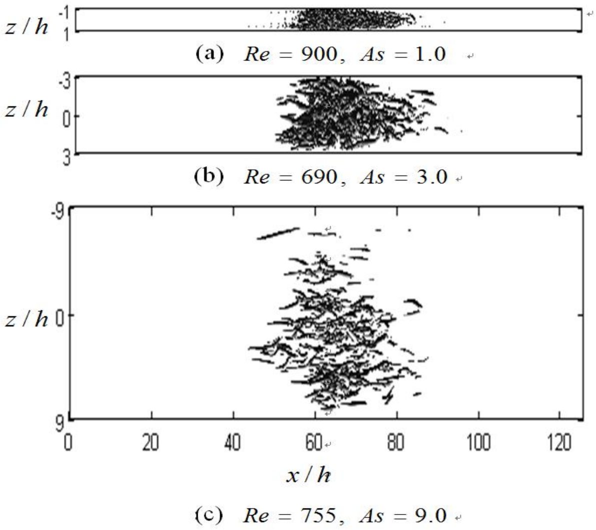


Fig. 3 Three-dimensional representation of vortical structures visualized by the isosurface of Q around the marginal Reynolds number.

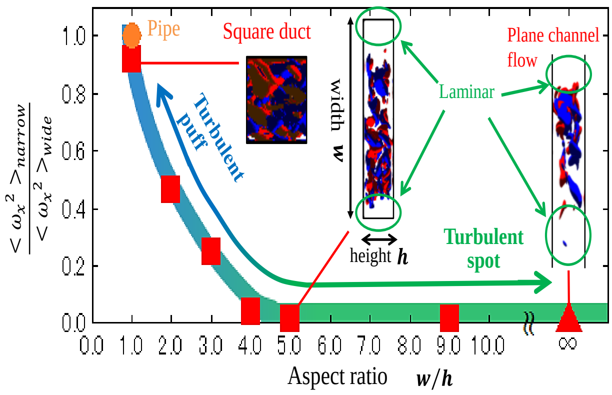


Fig. 4. The magnitude of the vorticity near the sidewall as the function of the aspect ratio.

Fig. 5 shows that the localized turbulent structures oblique and become turbulent stripe structure for $As \geqq 4.0$.

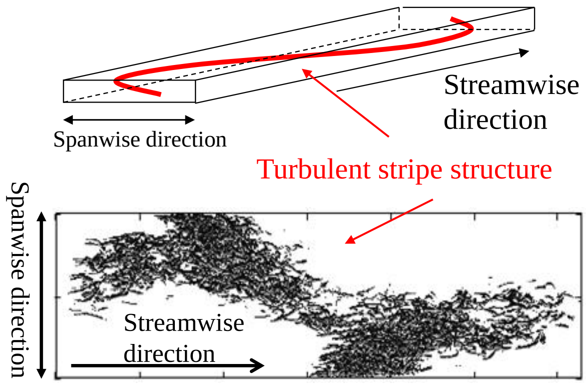


Fig. 5 The turbulent stripe structure.

Fig. 6 shows the dependence of marginal Reynolds numbers on As of rectangular duct. Note that As = 1 corresponds to the square duct, whereas $As \to \infty$ corresponds to the plane channel flow. It is observed in this figure that, for $As \geqq 4.0$, the marginal Reynolds number in the rectangular duct flow coincides with that in the plane channel flow.

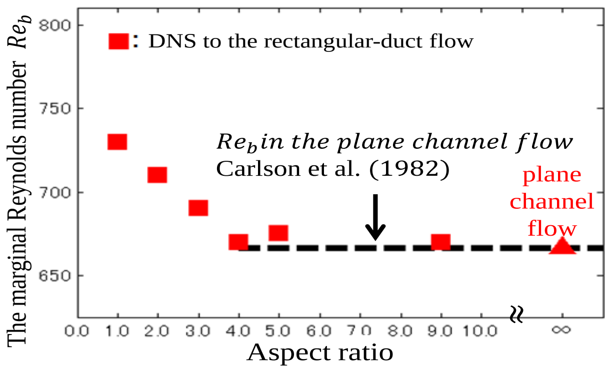


Fig. 6. The marginal Reynolds numbers as the function of the aspect ratio.

4 Conclusion

We have conducted the direct numerical simulations of turbulence in a rectangular duct at transitional Reynolds numbers (between turbulent and laminar flows), and revealed the dependence of the localized turbulent structures on the aspect ratio As of the cross-section of the duct. When $As < 4$, the structure is puff similarly to the flow in a square duct; whereas when $As > 4$ it becomes spot like in the plane channel turbulence. The dependence of the marginal Reynolds number on As is also shown to be consistent with these observations.

References

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