Estimation of Point Normals from Positional Relationship of Three-Dimensional Scattered Point Data

Taku Itoh[1]

[1]Faculty of Science and Technology, Seikei University, Japan

1 Introduction

The surface reconstruction problem for three-dimensional (3D) scattered point data derived from 3D range scanners has been investigated in fields such as computer graphics (CG) and computer-aided design (CAD) [1, 2]. By using an implicit function-based method, the surface reconstruction problem can be solved. However, it must be noted here that, for generation of an implicit function in general domain, point normals on each of given points may be required. For example, the multi-level partition of unity implicits (MPU) method [2], which is a typical method for generating an implicit function, requires the point normals. However, there are some cases that point data do not contain normal data. In this case, surface reconstruction from 3D scattered point data is not easy. Hence, development of an estimation method of point normals from complicated point data is desired.

The purpose of the present study is to propose an estimation method of point normals from a positional relationship of given point data.

2 Roughly-Generated Scalar-Valued Function

For a constant data value surface, it is well known that the gradient vector is normal to the surface. Therefore we can estimate point normals from given point data if a scalar-valued function is generated. Since an implicit function is one of the scalar-valued functions, we consider generation of an implicit function $f(\boldsymbol{p})$ from given point data $\mathcal{P} = \{\boldsymbol{p}_1, \boldsymbol{p}_2, \ldots, \boldsymbol{p}_n\}$ that do not contain the point normals. Here, n is the number of given points.

To generate an implicit function, constraint points $\hat{\mathcal{P}} = \{\boldsymbol{p}_{n+1}, \boldsymbol{p}_{n+2}, \ldots, \boldsymbol{p}_{n+m}\}$, together with scalar height values $\mathcal{H} = \{h_{n+1}, h_{n+2}, \ldots, h_{n+m}\}$ have to be placed. Here, m is the number of constraint points. Usually, $\hat{\mathcal{P}}$ and $\mathcal{H}$ are set by using point normals. However, we assume that given point data do not have point normals. Hence, we use the method described in Ref. [3] for setting $\hat{\mathcal{P}}$ and $\mathcal{H}$. By using $\mathcal{P}$, $\hat{\mathcal{P}}$ and $\mathcal{H}$, an implicit function $f(\boldsymbol{p})$ can roughly be generated [3].

The difference between the proposed method and the method described in Ref. [3] is modification procedures of estimated normals that are generated as $-\nabla f(\boldsymbol{p})/|\nabla f(\boldsymbol{p})|$. Thus, the proposed modification procedures of estimated normals are described in the next section.

3 Modification of Estimated Normals

A generated surface by the Delaunay tetrahedralization with the estimated normals occasionally has redundant triangles. The redundant triangles can be decreased by re-

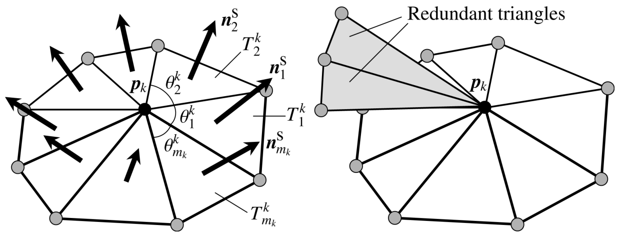

Fig. 1. Left: Triangles around $\boldsymbol{p}_k$ without redundant triangles. Here, $\boldsymbol{n}_i^{\mathrm{s}}$ is the surface normal of T_i^k, where T_i^k is the ith triangle around $\boldsymbol{p}_k$, and θ_i^k is the angle of corner of T_i^k. In addition, m_k is the number of triangles around $\boldsymbol{p}_k$. Right: Triangles around $\boldsymbol{p}_k$ with redundant triangles.

execution of the Delaunay tetrahedralization with more accurate normals. In this section, we present a method for modification of the estimated normals.

3.1 Estimated Point Normals Generated by Average of Surface Normals

For modification of the kth normal $\boldsymbol{n}_k$, the triangles around $\boldsymbol{p}_k$ are first found. Let these triangles be $\mathcal{T}_k = \{T_1^k, T_2^k, \ldots, T_{m_k}^k\}$, where m_k is the number of triangles around $\boldsymbol{p}_k$. After that, the sum of angles $\{\theta_1^k, \theta_2^k, \ldots, \theta_{m_k}^k\}$ that are corner of T_i^k around $\boldsymbol{p}_k$ is calculated. Here, we assume that triangles of $\mathcal{T}_k$ are approximately placed on a plane surface locally. Under this assumption, if redundant triangles do not exist in $\mathcal{T}_k$, the sum of angles of these triangles may be close to 2π. An ideal case is illustrated in the left of Fig. 1. In this case, the kth point normals $\boldsymbol{n}_k$ is estimated as the average of $\{\boldsymbol{n}_1^{\mathrm{s}}, \boldsymbol{n}_2^{\mathrm{s}}, \ldots, \boldsymbol{n}_{m_k}^{\mathrm{s}}\}$, where $\boldsymbol{n}_i^{\mathrm{s}}$ is the surface normal of T_i^k. However, in case that the sum of $\{\boldsymbol{n}_1^{\mathrm{s}}, \boldsymbol{n}_2^{\mathrm{s}}, \ldots, \boldsymbol{n}_{m_k}^{\mathrm{s}}\}$ nearly equals to zero, the $\boldsymbol{n}_k$ can not be estimated by the average of surface normals. In this case, the $\boldsymbol{n}_k$ is estimated by other procedures. In addition, the sum of angles is not close to 2π, the $\boldsymbol{n}_k$ is also estimated by other procedures.

3.2 Recognition of Redundant Triangles

If the sum of angles is greater than 2π, some redundant triangles may exist around $\boldsymbol{p}_k$ (see the right of Fig. 1). In this case, the kth point normals $\boldsymbol{n}_k$ is estimated after recognition of redundant triangles. For recognizing redundant triangles, we first obtain $\{s_1^k, s_2^k, \ldots, s_{m_k}^k\}$, where s_i^k is the maximum length of three sides of T_i^k, since redundant triangles tend to have large sides in comparison with other triangles (see experimental result shown in Fig. 2b). After that, $\{T_1^k, T_2^k, \ldots, T_{m_k}^k\}$ are renumbered in ascending order of $\{s_1^k, s_2^k, \ldots, s_{m_k}^k\}$. After renumbering, the sum of the angles $\{\theta_1^k, \theta_2^k, \dots, \theta_j^k\}$ and the sum of $\{\boldsymbol{n}_1^{\mathrm{s}}, \boldsymbol{n}_2^{\mathrm{s}}, \dots, \boldsymbol{n}_j^{\mathrm{s}}\}$ are calculated, where j is determined so that $\left|2\pi - \sum_{i=1}^{j} \theta_i^k\right| < \varepsilon$ is satisfied. Here, we set $\varepsilon = 0.1$. Note that j occasionally does not exist. If j is found, we consider $\{T_{j+1}^k, T_{j+2}^k, \dots, T_{m_k}^k\}$ as redundant triangles, and $\boldsymbol{n}_k$ will be estimated by the average of $\{\boldsymbol{n}_1^{\mathrm{s}}, \boldsymbol{n}_2^{\mathrm{s}}, \dots, \boldsymbol{n}_j^{\mathrm{s}}\}$. If j is not found, we consider that recognition of redundant triangles is difficult. Hence, $\boldsymbol{n}_k$ will be estimated by a weighted average of other point normals. Note that, even if j is found, $\sum_{i=1}^{j} \boldsymbol{n}_i^{\mathrm{s}}$ is nearly equal to zero infrequently. In this case, $\boldsymbol{n}_k$ will also be estimated by the weighted average.

3.3 Estimated Normals by Weighted Average

After the above procedures are finished, some point normals do not estimated. For this case, we employ a weighted average of other point normals as follows:

$$\hat{\boldsymbol{n}}_k = \frac{\sum_{i=1}^{N_{\mathrm{np}}^k} w(r_{ki}) \boldsymbol{n}_i^k}{\sum_{i=1}^{N_{\mathrm{np}}^k} w(r_{ki})}, \quad \boldsymbol{n}_k = \frac{\hat{\boldsymbol{n}}_k}{\|\hat{\boldsymbol{n}}_k\|_2},$$

where $r_{ki} = \|\boldsymbol{p}_i^k - \boldsymbol{p}_k\|_2$, and $w(r)$ is a weight function,

$$w(r) = \begin{cases} 1 - 6\left(\frac{r}{R}\right)^2 + 8\left(\frac{r}{R}\right)^3 - 3\left(\frac{r}{R}\right)^4, & \text{for } r \le R, \\ 0, & \text{for } r > R, \end{cases}$$

where R is the radius of the support. In addition, the collection of the N_{np}^k nearest points from $\boldsymbol{p}_k$ are $\{\boldsymbol{p}_1^k, \boldsymbol{p}_2^k, \dots, \boldsymbol{p}_{N_{\mathrm{np}}^k}^k\}$, and the estimated point normals $\{\boldsymbol{n}_1^k, \boldsymbol{n}_2^k, \dots, \boldsymbol{n}_{N_{\mathrm{np}}^k}^k\}$ correspond to these points, respectively. Note that $\{\boldsymbol{p}_1^k, \boldsymbol{p}_2^k, \dots, \boldsymbol{p}_{N_{\mathrm{np}}^k}^k\}$ are selected from the points that already have the estimated point normals after finishing the flow chart of Fig. 1. Here, N_{np}^k is determined as the number of points that are contained in the k-th radius of the support R^k, where R^k is obtained by $R^k = R^k + \alpha R_{\mathrm{ini}}$ that is iterated until $N_{\mathrm{np}}^k \ge N_{\min}$. We set $\alpha = 0.1$ and $N_{\min} = 40$. In addition, $R_{\mathrm{ini}} = \beta_R \max(x_{\max} - x_{\min}, y_{\max} - y_{\min}, z_{\max} - z_{\min})$, where β_R is a small positive value like 0.05.

4 Experiments

In this section, some experiments are conducted to demonstrate the proposed method by using the data illustrated in Fig. 2a. First, ℓ denotes the number of repetitions of the normal modification procedures described in Sect. 3. The initial surface is shown in Fig. 2b. For the initial surfaces, we set $\ell = 0$.

In Fig. 2b, obvious redundant triangles cover an expected surface. Figures. 2c, 2d and 2e show the reconstructed results with $\ell = 3, 7$ and 14, respectively. We see from Figs. 2c, 2d and 2e that the redundant triangles of Fig. 2b are gradually decreased. In addition, an implicit surface $f(\boldsymbol{p}) = 0$ that is generated from $\mathcal{P}$ and the estimated normals in Fig. 2e is shown in Fig. 2f. Here, we investigate the accuracy of the implicit function $f(\boldsymbol{p})$ for generating Fig. 2f. To this end, we employ the root-mean-square (RMS) error $\varepsilon_{\mathrm{RMS}} = \sqrt{\sum_{i=1}^{n} [f(\boldsymbol{p}_i)]^2 / n}$. The result is $\varepsilon_{\mathrm{RMS}} = 2.2 \times 10^{-5}$. From this result, since the reconstructed result of Fig. 2f is visually sufficient, we consider that the implicit function has sufficient accuracy for visualization. Therefore we consider that accurate normals can be estimated by using the proposed method.

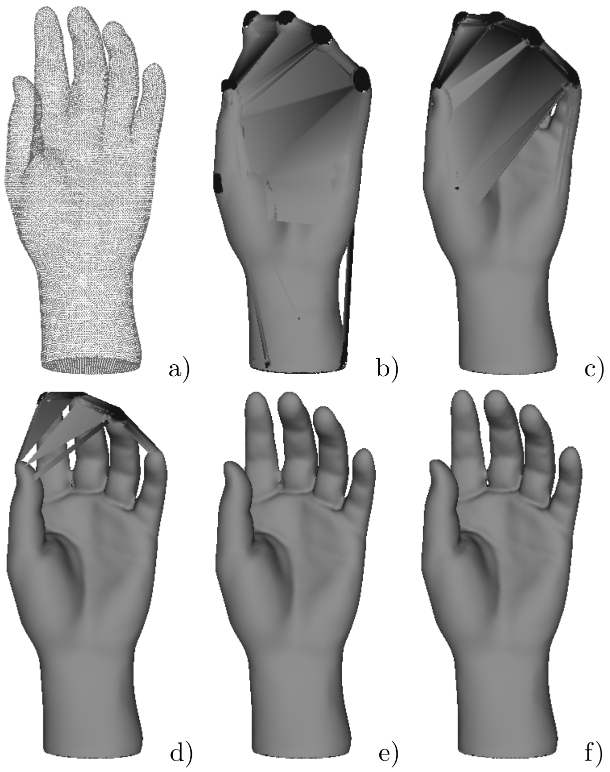

Fig. 2. Results of the surface reconstruction from the given point data shown in a). Here, b) is an initial surface ($\ell = 0$). Obvious redundant triangles cover an expected surface. In addition, c), d) and e) are reconstructed results with $\ell = 3, 7$ and 14 respectively. f) is the implicit surface generated from $\mathcal{P}$ and the estimated normals in Fig. 2e.

5 Conclusion

An estimation method of point normals from a positional relationship of given point data has been proposed. In the experiments, the performance of the proposed method has been investigated by using "Hand" model that has concave aspects.

The experiments show that, by using the proposed method, redundant triangles are gradually decreased and the reconstructed result can finally be obtained. In addition, the reconstructed result has sufficient accuracy for visualization. Therefore we consider that the point normals can be estimated from a positional relationship of point data by using the proposed method.

Acknowledgement

We would like to thank the FarField Technology Ltd for the Hand model.

References

[1] T. Taniguchi and K. Moriwaki, *Automatic Mesh Generation for 3D FEM - Robust Delaunay Triangulation (in Japanese)*. Tokyo: Morikita, 2006.

[2] Y. Ohtake, A. Belyaev, M. Alexa, G. Turk, and H.-P. Seidel, "Multi-level partition of unity implicits," *ACM Trans. on Graph.*, vol. 22, no. 3, pp. 463–470, 2003.

[3] T. Itoh, "A method of boundary estimation from 3D scattered point data without normals by implicit function and Delaunay tetrahedralization," *Journal of Visualization*, vol. 14, no. 4, pp. 381–391, 2011.