

# Effective Constitutive Characteristics of Racemic Mixture of Thin-Wire Helices

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## 1 Introduction

The artificial media have been of great interest in the communities of electromagnetic wave engineering including the fields of microwaves, milli-meter waves, optics etc. These are the fabricated aggregation structures composed of artificial molecules i.e. small objects which are designed to show required macroscopic electromagnetic characteristics. The sizes and spacings between the objects are usually designed to be much smaller than the wavelength. Such artificial structures was studied first by K. F. Lindman in 1914 through 1922 [1] for the research on the dispersion characteristics of the optical activity of the artificial chiral media as the magnified scale models of the organic compounds. Research on artificial chiral media as well as that on artificial dielectrics and magnetic materials has been on-going through the golden years of the chiral media in '90s [2], [3]. The greatest concern on the artificial media has moved to the left-hand materials (LHM) with the term 'metamaterials' since J. B. Pendry *et al.* theoretically treated LHM [4], and D. R. Smith *et al.* demonstrated the negative index of refraction in a fabricated LHM in microwave regions at the end of the '90s [5],[6]. On the other hand, chiral media have been of great interest in the researchers of carbon nano coils (CNCs) and micro coils (CMCs) especially since the synthesis of CMCs in high yield content was discovered and the fine absorption characteristics at frequencies of tens through hundreds GHz was shown [7],[8]. The effective constitutive parameters of isotropic distributions of chiral mixtures of thin-wire helices have been analyzed by the quasi-static Lorentz' approach under the conditions that helices with an identical dimension cluster in groups [9], and that helices in clusters have elastic deviations [10],[11] as the models of isotropic chiral mixture of carbon coils. The bulks of CMCs synthesized for a large amount however are usually composed of almost the same amount of right- and left-handed coils i.e. racemic mixture and are macroscopically achiral. The method to separate the right- and left-handed coils has been explored and not yet discovered. Recent development of material sciences has made it possible to realize the array of clusters of carbon coils with identical orientation, and such array structures would bear the possibility of new bianisotropic media [2]. It is reported that the multipole moments to the order of electric quadrupole and magnetic dipole are necessary to explain the anisotropic chirality in natural materials [12],[13]. In most of the analyses for bianisotropic cases however, the structures of inclusions do not require the order of multipole moments higher than that of dipoles [14], [15], or the contributions of multipole moments themselves are not shown for different structural parameters [16], [17]. Chiral media composed of single-helix particles or particles of pairs of helices were analyzed and the contributions of quadrupole moments in the constitutive characteristics were investigated [18]

In this work, the effective constitutive characteristics are analyzed for the isotropic and anisotropic racemic mixture of metallic thin-wire helices. Inclusion particles are composed of one or two metallic thin-wire helices in free space. The sizes and spacings between helices are assumed to be small compared to

the wavelength. The quasi-static Lorentz' approach with fine evaluation of interaction between particles [19] is combined with the method of moments with thin-wire approximation to calculate the effective parameters. The contributions of the electric quadrupole moments in the determination of the effective constitutive parameters are also investigated.

## 2 Analysis

Both isotropic and anisotropic distributions of particles are assumed. For the anisotropic cases, three-dimensional periodic array with the periodicities along the Cartesian coordinates  $x$ ,  $y$  and  $z$  being  $d_x$ ,  $d_y$  and  $d_z$  respectively is assumed. The sizes and the spacings of the particles are assumed to be much smaller than the wavelength for the quasi-static approximation. The permittivity and the permeability for free space and time dependence are assumed to be  $(\epsilon_0, \mu_0)$  and  $\exp(j\omega t)$  respectively. Each particle is assumed to be composed of a right-handed metallic thin-wire single helix or a skew pair of

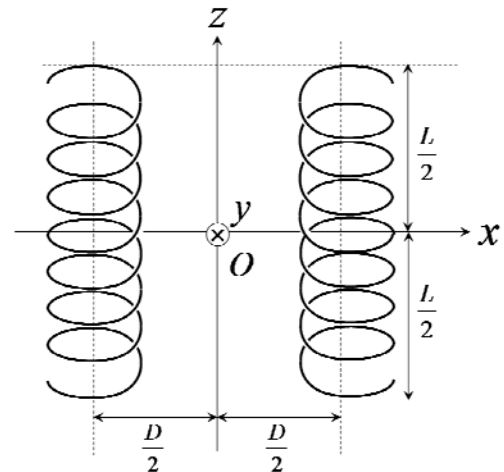


Figure 1: A pair of parallel helices

right and left-handed metallic thin-wire helices with number of turns  $T$ , height  $L$ , radius  $A$ , pitch  $P$ , wire radius  $W$ , and wire length  $l$ . The axis of the single helix is oriented in  $z$  direction. As for the pairs of helices, parallel pairs are also assumed as the special case of skew ones. Figure 1 shows the parallel pair of helices where the axes of two helices with distance  $D$  are parallel. The end of the wire of a single-helix particle or the right helix of a pair of helices (Fig. 1) in the  $z < 0$  region are assumed to be located in the position  $(A \cos \varphi, A \sin \varphi, -L/2)$  where the angle  $\varphi$  in the plane parallel to  $xy$  plane is defined in the clockwise direction on the  $z$  axis from the  $xz$  plane. The skew cases can be realized by rotating

the right helix clockwise on the  $x$  axis for angle  $\theta$ . The quasi-static Lorentz' approach with fine evaluation of interaction between particles [19] is combined with the method of moments to calculate the effective parameters. In the method of moments only the axial components of currents on the wire surfaces are considered and the circumferential variations are neglected for the thin wire approximation.

### 3 Numerical Investigations

Calculations are performed under the conditions that  $l=25.0(\text{mm})$ ,  $L=4.0(\text{mm})$ ,  $P=L/T$  and  $W=0.07(\text{mm})$ , where the periodicities of the three-dimensional array are chosen as  $d_x=d_y=4.0A$ ,  $d_z=2.0L$  for the single-helix cases, and  $d_x=d_y=2.5U$ ,  $d_z=2.5L$  for the cases of pairs of helices. Results will be shown in the full manuscript and in the presentation.

### 4 Conclusion

The effective constitutive characteristics are analyzed for the isotropic and anisotropic racemic mixture of metallic thin-wire helices. Inclusion particles are composed of one or two metallic thin-wire helices in free space. The sizes and spacings between helices are assumed to be small compared to the wavelength. The quasi-static Lorentz' approach with fine evaluation of interaction between particles is combined with the method of moments with thin-wire approximation to calculate the effective parameters. The contributions of the electric quadrupole moments in the determination of the effective constitutive parameters are also investigated.

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