Simulation of collisionless plasma with the Vlasov method

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1 Introduction

Space plasma is a collisionless, multi-scale, and highly nonlinear medium. There are numerous types of self-consistent computer simulations that treat space plasma according to various approximations. The global-scale dynamics are commonly described by magneto-hydro-dynamic (MHD), Hall-MHD and multi-fluid models, while electron-scale processes are described by the kinetic model, i.e., the Maxwell equations and either the Newton-Lorentz equation for charged particles or the Vlasov (collisionless Boltzmann) equation. Hybrid methods treat ions as particles and electrons as a fluid for ion-scale processes.

Conventionally, MHD simulations have been used for numerical modeling of global-scale problems such as magnetospheres of stars and planets. However, the MHD simulations need diffusion coefficients, which are essentially due to kinetic processes that are eliminated in the framework of the MHD approximation. Recent high-resolution in-situ observations have also suggested that fluid scale and kinetic scale in space plasma are strongly coupled with each other, which is called cross-scale coupling. To understand the cross-scale coupling in space plasma, it is important to include full kinetics in global-scale simulations, which is the goal of this study.

2 Overview of Numerical Schemes

The Vlasov model solves the kinetics equations of space plasma, i.e., the Maxwell equations (1) and the Vlasov (collisionless Boltzmann) equation (2),

$$\left.\begin{array}{c} \nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \dfrac{1}{c^2}\dfrac{\partial \mathbf{E}}{\partial t} \\ \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \end{array}\right\} \qquad (1)$$

$$\frac{\partial f_s}{\partial t} + \mathbf{v}\frac{\partial f_s}{\partial \mathbf{r}} + \frac{q_s}{m_s}\left[\mathbf{E} + \mathbf{v} \times \mathbf{B}\right]\frac{\partial f_s}{\partial \mathbf{v}} = 0 \qquad (2)$$

where $\mathbf{E}$, $\mathbf{B}$, $\mathbf{J}$, ρ, μ_0, ϵ_0 and c represent electric field, magnetic field, current density, charge density, magnetic permeability, dielectric constant and light speed, respectively. The Vlasov equation (2) describes the development of the distribution functions by the electromagnetic (Lorentz) force, with the collision term in the right hand side set to be zero. The distribution function $f_s(\mathbf{r}, \mathbf{v}, t)$ is defined in position-velocity phase space with the subscript s being the species of singly-charged particles (e.g., $s = i, e$ for ions and electrons, respectively). The Maxwell equations and the Newton-Lorentz equations or the Vlasov equation are coupled with each other via the current density $\mathbf{J}$ that satisfies the continuity equation for charge

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{J} = 0 \qquad (3)$$

These equations are regarded as the "first principle" of the collisionless plasma.

The Vlasov equation (2) consists of two advection equations with a constant advection velocity and a rotation equation by a centripetal force without diffusion terms. To simplify the numerical time-integration of the Vlasov equation, we adopt a modified version of the operator splitting [1],

$$\frac{\partial f_s}{\partial t} + \mathbf{v}\frac{\partial f_s}{\partial \mathbf{r}} = 0 \qquad (4)$$

$$\frac{\partial f_s}{\partial t} + \frac{q_s}{m_s}\mathbf{E}\frac{\partial f_s}{\partial \mathbf{v}} = 0 \qquad (5)$$

$$\frac{\partial f_s}{\partial t} + \frac{q_s}{m_s}\left[\mathbf{v} \times \mathbf{B}\right]\frac{\partial f_s}{\partial \mathbf{v}} = 0 \qquad (6)$$

Equations (4) and (5) are scalar (linear) advection equations in which $\mathbf{v}$ and $\mathbf{E}$ are independent of $\mathbf{r}$ and $\mathbf{v}$, respectively. We adopt a multidimensional conservative semi-Lagrangian scheme [1] for solving the multidimensional advection equations. In the full electromagnetic method, it is essential to use conservative schemes for satisfying the continuity equation for charge. With the multidimensional conservative semi-Lagrangian scheme, the continuity equation for charge (3) is exactly satiesfied. In the present study, we compute the numerical flux by using the multi-dimensional advection scheme [1] with a positive, non-oscillatory and conservative limiter [2, 3] for stable time-integration of advection equations. Equation (6), on the other hand, is a multi-dimensional rotation equation which follows a circular motion of a profile at constant speed by a centripetal force. For stable rotation of the profile on the Cartesian grid system, the "back-substitution" technique [4] is applied. In addition, Maxwell's equations are solved by the implicit Finite Difference Time Domain (FDTD) method.

The velocity distribution function has both configuration-space and velocity-space dimensions, and defined as a hyper-dimensional (>3D) array, There are some additional communications overhead in parallelizing over the velocity-space dimensions since a reduction operation is required to compute the charge and current densities (the zeroth and first moments) at a given point in configuration space. Thus we adopt the "domain decomposition" only in configuration space, where the distribution functions and electromagnetic fields are decomposed over the configuration-space dimensions. This involves the exchange of ghost values for the distribution function and electromagnetic field data along boundaries of each procession element. The non-oscillatory and conservative scheme [2, 3] uses six grids for numerical interpolation, and three ghost grids are exchanged by using the "`Mpi_Sendrecv()`" subroutine in the standard message passing interface (MPI) library for simplicity and portability [5]. Note that the code allows thread parallelization over the velocity-space dimensions via OpenMP.
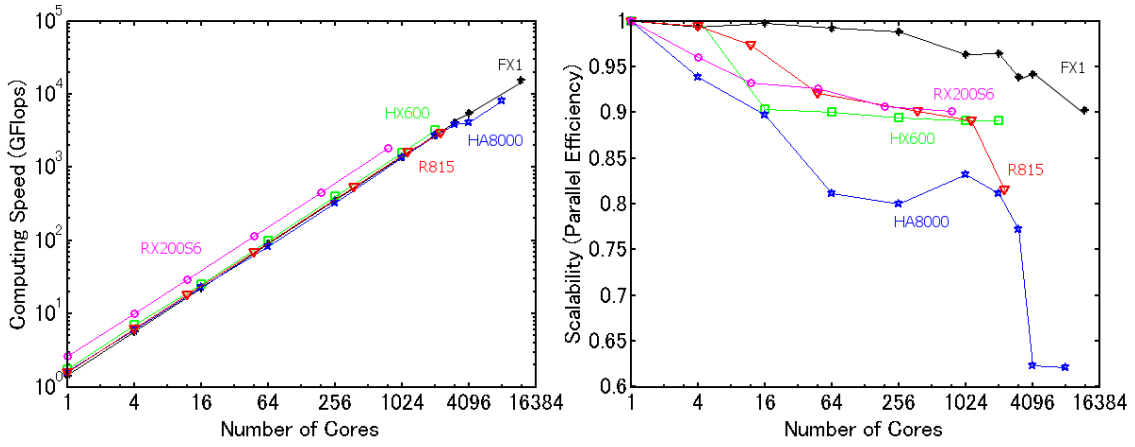
Fig. 1. Performance of different supercomputer systems with scalar processors. (left) Computational speed and (right) Scalability as a function of the number of cores. The star, asterisk, square, circe, and downward-pointing-triangle marks represent the results on the HA8000, FX1, HX600, RX200S6, and R815. respectively.

3 Performance Evaluation

We conduct the performance measurement test of our parallel Vlasov code with a phase-space grid of $(N_{v_x}, N_{v_y}, N_{v_z}, N_x, N_y) = (30, 30, 30, 40, 20)$ on one core, which corresponds to a weak-scaling test with 1GB/core. The tests were completed on various scalar-type parallel supercomputers, Hitachi HA8000 at the University of Tokyo, Fujitsu FX1 at Nagoya University and JAXA, Fujitsu HX600 at Nagoya University, Fujitsu RX200S6 at Kyushu University, and DELL PowerEdge R815 at Solar-Terrestrial Environment Laboratory, Nagoya University. Figure 1 shows the computational performance on these systems. As seen, we obtained a high scalability of over 80% with 1,000 cores. However, the scalability becomes worse when the all computational resources (3,072 and 12,032 cores on FX1, and 2,304 cores on R815) are used. It should be noted that the internode-connection device of the HA8000 system is Myrinet-10G, and the scalability becomes worse with more than 1,024 cores because of the network bandwidth capacity.

4 Conclusion

For studying multi-scale processes in space plasma, computer simulations with the first-principle (kinetic) model are essential. We develop numerical schemes for Vlasov simulations for practical use on currently-existing supercomputer systems. The weak-scaling benchmark test shows that our parallel Vlasov code achievs a high scalability. Currently, we use 256-1024 cores for parallel computations and apply the present parallel Vlasov code to "global" simulation on the interaction between solar/stellar winds and a weakly-magnetized small body (Fig.2).
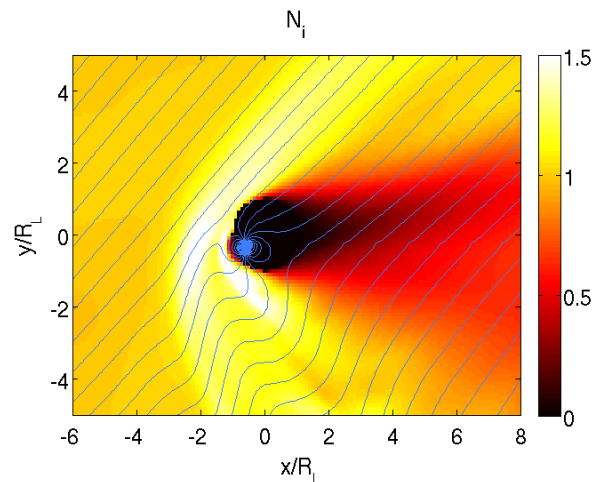
Acknowledgement

Fig. 2. Structures of bow shock and wake tail formed as a result of the interaction between a weakly-magnetized object and the solar wind. Spatial profile of ion density together with the configuration of magnetic field lines obtained by a global Vlasov simulation.

References

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