Convergence property of Krylov subspace methods for asymmetric linear system on fusion plasma simulation code GT5D

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1. Introduction

The gyrokinetic toroidal five-dimensional Eulerian code GT5D [1] is one of the first-principle simulation codes for the fusion plasma turbulence. The code has been parallelized and ported on some massively parallel platforms, and then, it has been confirmed that the code achieves good parallel efficiency on them. In GT5D code, the second-order additive semi-implicit Runge-Kutta (ASIRK) method [2] is adopted for the time-evolution simulation. Therefore, we need to solve the linear equations, whose coefficient matrices are asymmetric. In the original code, the generalized conjugate residual (GCR) method without preconditioning has been so far utilized. Thus, it is expected that the convergence property can be improved by substituting a more suitable preconditioned solver. In this research, we examine some preconditioned Krylov subspace methods and measure their convergence properties. Moreover, we evaluate the parallel performance of the solvers.

2. Convergence Property of Krylov Subspace Methods

2.1 Preconditioners

Here, we discuss the effect of preconditioning for the linear equations. At first, we examine the convergence property of the incomplete LU (ILU) decomposition. The ILU decomposition can improve the convergence property for various linear equations. However, when we employ the ILU decomposition, the convergence property in GT5D code becomes worse. The reason is that the ILU decomposition does not approximate the coefficient matrix well, since the diagonal elements of the coefficient matrix are small as compared with the off-diagonal elements.

Next, we propose the preconditioning based on the SOR iteration method. The preconditioning is executed according to the following formula

$$x_i^{n+1} = x_i^n + \frac{\omega}{m_{ii}}\left(b_i - \sum_{j=1}^{i-1} m_{ij} x_j^{n+1} - \sum_{j=i}^{N} m_{ij} x_j^n\right),$$

where m_{ij} is the (i, j)-element of preconditioner M, which is an approximate matrix of the coefficient matrix. Table 1 shows the effect of the preconditioning based on the SOR iteration. The problem size is 60×60×32×28 in *xyzv*-space. Here, since the coefficient matrix of the linear equation is given by 17 stencil grids in 4-dimensional space, the preconditioner can be composed of the elements in each direction. Therefore, we compare the convergence property for the some preconditioners covering all or partial dimensions. The result shows that the preconditioner covering all directions is the best and the effect of the *v*-direction does not almost exist.

Table 1. Relationship between the dimensions covered by the preconditioner M and the number of iterations for preconditioned GCR method.

Preconditioner (covering dimensions)	ω	Number of iterations
None	—	466
x, y-direction	—	Not converge
x, z –direction	0.047	289
y, z –direction	0.043	282
z, v –direction	0.049	347
x –direction	—	Not converge
z –direction	0.049	354
v –direction	1.8	466
x, y, z, v –direction (all dimensions)	0.054	181

2.2 Convergence Property of Preconditioned Krylov Subspace Methods

Here, we examine the effects of the preconditioners covering all or some dimendions for GCR method, Bi-CGstab method, and GMRES method.

- **GCR method**

We show the convergence property in Figure 1. The result demonstrates that the number of iterations decreases from 466 to 181 by utilizing the preconditioning.

- **Bi-CGstab method**

Figure 2 shows the convergence property of the Bi-CGstab method. The convergence of the Bi-CGstab method without preconditioning is very slow (the method can converge by about 5000 iterations). The preconditioning yields a large convergence accelaration.

- **GMRES method**

Figure 3 shows the convergence property of the Bi-CGstab method. Here, the restart parameter is 4. The number of iterations decreases to about 1/4 by utilizing the preconditioning.

These results demonstrate that the preconditioning based on the SOR iteration method is suitable for this equation.

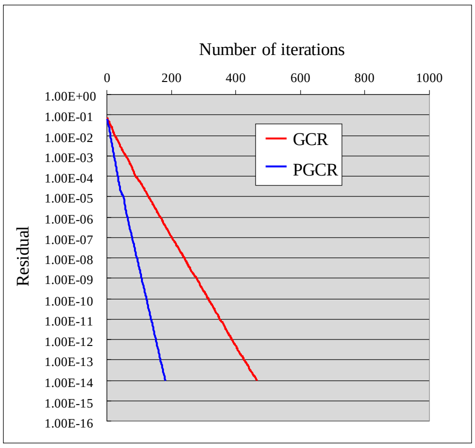

Figure 1. Convergence property of GCR method.

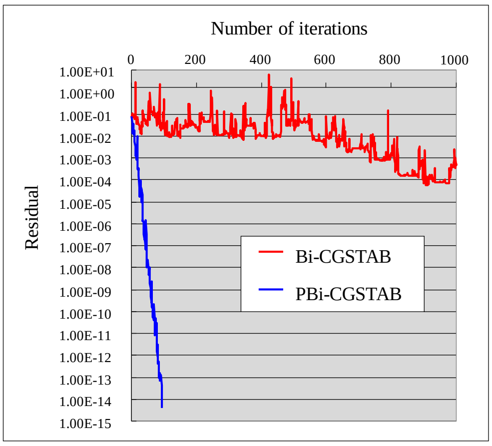

Figure 2. Convergence property of Bi-CGstab method. The convergence of the method without preconditioning is very slow. On the other hand, the preconditioned method can solve the equation very fast.

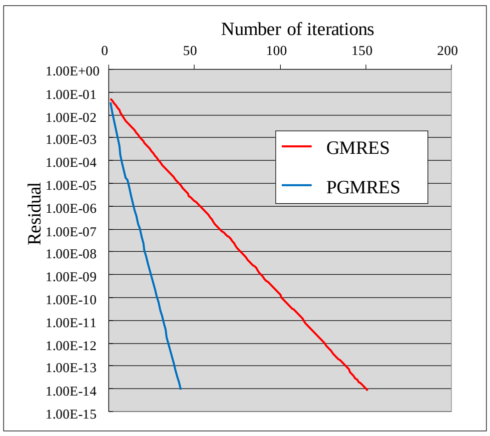

Figure 3. Convergence property of GMRES method. The restart parameter is 4. The number of iterations decreases to about 1/4 by utilizing the preconditioning.

3. Numerical Experiments

We examine the parallel performance of the preconditioned solvers on 256 cores of Fujitsu BX900 in Japan Atomic Energy Agency. The problem size is 160×160×32×128 in *xyzv*-space. Since table 1 shows that the effect of the preconditioning for the *v*-direction does not almost exist, we employ the preconditioners covering the all directions and *x,y,z* directions. We show the number of the iterations and the elapsed times in Table 2. The result indicates that the effects of both preconditioners are almost the same. Since calculation cost of the *x,y,z*-direction preconditioner is less than that of all-direction one, the *x,y,z*-direction preconditioner can be performed faster.

Table 2. Number of iterations and Elapsed time on 256 cores of BX900. P and P_{xyz} means all-direction preconditioner and *x,y,z*-direction one, respectively. Since Bi-CGstab method without preconditioning does not converge within 1000 iterations, its result is omitted.

method	ω	Number of iterations	Elapsed time (sec)
GCR	—	641	110.7195
PGCR	0.037	260	107.2379
P_{xyz}GCR	0.040	256	104.7527
PBi-CGSTAB	0.038	144	110.8538
P_{xyz}Bi-CGSTAB	0.038	141	97.9747
GMRES	—	210	208.4488
PGMRES	0.047	60	130.5454
P_{xyz}GMRES	0.047	60	118.5158

4. Conclusion

We examined the convergence property of some Krylov subspace methods for the linear equation on GT5D code. The result showed that the Bi-CGstab method with the preconditioning based on the SOR iteration is the fastest solver.

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Reference

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