Influence of Turbulent Droplet Clustering on Microwave Scattering

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1 Introduction

Clouds play crucial roles in the heat and water systems of the Earth. A large number of observations have been conducted to understand the cloud physics. The radar observation is one of the powerful tools since it can provide two- or three-dimensional data regarding the cloud microphysical properties such as the cloud water mixing ratio and effective droplet radius [1, 2]. In the radar observation, microwave is transmitted from an antenna toward a target cloud and the reflected microwave is received and analysed. The relation between the transmitted power, P_t, and the received power, P_r, of the microwave is given as the following radar equation:

$$P_r = \frac{P_t G^2 k_{MW}{}^2 |K|^2 V}{4^5 R^4} Z, \tag{1}$$

where G is the antenna gain, k_{MW} the wavenumber of microwave, R the distance between the antenna and the cloud, K the dielectric coefficient of a water droplet, and V the measurement volume. Z is the radar reflectivity factor. An important point is that Z is dependent on the cloud microphysical properties. This means that the properties can be estimated from Z under some assumptions: homogeneity and randomness of cloud droplet distribution are usually assumed. However, cloud turbulence forms inhomogeneous spatial distribution of droplets, which is often referred to turbulent clustering or preferential concentration [3, 4, 5]. Turbulent clustering obviously contradicts the homogeneity assumption for the analysis of Z. This may cause significant observational error.

This study, therefore, aims to investigate the effect of turbulent clustering of droplets on the radar reflectivity factor, i.e. microwave scattering. At first, a radar reflectivity factor model, which takes account of turbulent clustering, is developed based on the data of a three-dimensional direct numerical simulation (DNS) of particle-laden isotropic turbulence. The model is then applied to high-resolution cloud-simulation data in order to estimate the possible error due to turbulent clustering in a radar observation.

2 DNS for Isotropic Turbulence

Turbulent air flow was simulated by using a pseudo-spectral method based on the Fourier–Galerkin method. The detail of numerical method is described in Onishi et al. [7]. Statistically steady-state turbulence was formed by applying an external force to maintain the intensity of large-scale eddy.

Droplet motions were tracked by the Lagrangian method. The governing equation of the motion is $dv_i/dt = -(v_i - u_i)/\tau_p$, where u_i is the fluid velocity, v_i the droplet velocity, and τ_p the droplet relaxation time. Droplets were assumed as Stokes particles, whose relaxation time τ_p is given as $\tau_p = (\rho_p/\rho_g)(2r^2/9\nu)$, where r is the droplet radius, ρ_p the water density, ρ_g the air density, and ν the kinematic viscosity. Effects of gravity and collisions between droplets were neglected [8].

The computational domain was set to a cube with length of $2\pi L_0$, where L_0 is the representative length. Periodic boundary conditions were applied in all three directions. Table 1 shows computational conditions for air turbulence and statistical properties obtained from DNS. Droplet radius was varied so that the Stokes number, St, which is the ratio of the droplet relaxation time, τ_p, to the Kolmogorov time scale, τ_η, becomes 0.05, 0.1, 0.2, 0.5, 1.0, 2.0 and 5.0.

Table 1. Numerical conditions and flow properties. u_{rms} is the RMS value of velocity fluctuation, Re the Reynolds number defined as $Re = L_0 u_{rms}/\nu$, l_η the Kolmogorov scale, l_λ the Taylor's microscale, Re_λ the turbulent Reynolds number defined as $Re_\lambda = l_\lambda u_{rms}/\nu$.

L_0 [m]	u_{rms} [m/s]	Re	l_η [10^{-4} m]	l_λ [10^{-3} m]	Re_λ
0.020	0.32	427	2.65	6.45	152

3 Radar Reflectivity Factor

Radar reflectivity factor, Z, is calculated by the following equation [6, 9]:

$$Z = n_p (2r)^6 + \frac{2\pi^2 (2r)^6}{\kappa^2} E_{np}(\kappa), \tag{2}$$

where n_p is the droplet number density and κ is given as $\kappa = 2k_{MW}$. $E_{np}(k)$ is the power spectrum of droplet number density fluctuations. The second term in the right-hand side describes the increment of Z due to droplet clustering. That is, $E_{np}(k)$ represents the effect of turbulent clustering. Unfortunately, there has not been a widely accepted analytical model for $E_{np}(k)$ [10, 11, 12]. Thus, in this study, $E_{np}(k)$ was calculated from the DNS data as

$$E_{np}(k) = \frac{1}{\Delta k} \sum_{k-\frac{1}{2}\Delta k \le |\mathbf{k}| \le k+\frac{1}{2}\Delta k} \tilde{\Phi}(\mathbf{k}), \tag{3}$$

where $\tilde{\Phi}(\mathbf{k})$ is the spectral density function of droplet number density fluctuation given as

$$\frac{\tilde{\Phi}(\mathbf{k})}{\langle n_p \rangle^2 L_0{}^3} = \frac{1}{N_p{}^2} \left\langle \sum_{i=1}^{N_p} \exp(-i\mathbf{k}\cdot\mathbf{x}_{p,i}) \sum_{j=1, j\ne i}^{N_p} \exp(i\mathbf{k}\cdot\mathbf{x}_{p,j}) \right\rangle, \tag{4}$$

where $\mathbf{x}_{p,i}$ is the i-th droplet position vector, n_p the droplet number density, N_p the number of droplets. Brackets, $\langle\ \rangle$, represents the ensemble average. Eq. (4) was calculated for discrete wavenumber vector, $\mathbf{k}$, which satisfies $\mathbf{k} = (l/L_0, m/L_0, n/L_0)$ for arbitral integer numbers l, m and n.

4 Results and Discussion

4.1 Droplet Distributions in Turbulence

Fig. 1 shows the spatial distributions of droplets within the range of $0<z<4l_\eta$ for St=0.05, 0.2, 1.0 and 5.0. Turbulent clustering is clearly observed for St=1.0, and less clear for smaller and larger St. It is obvious that the cluster scale for St=5.0 is larger than that for St=1.0.

Fig. 2 shows the power spectra of droplets number density fluctuation, $E_{np}(k)$. The horizontal and vertical axes are normalized by the Kolmogorov scale, l_η, and the average number density, $\langle n_p \rangle$. $E_{np}(k)$ strongly depends on St, and does not show the −5/3 power law, which is often observed in a scalar concentration spectrum. This is because the fluctuation of

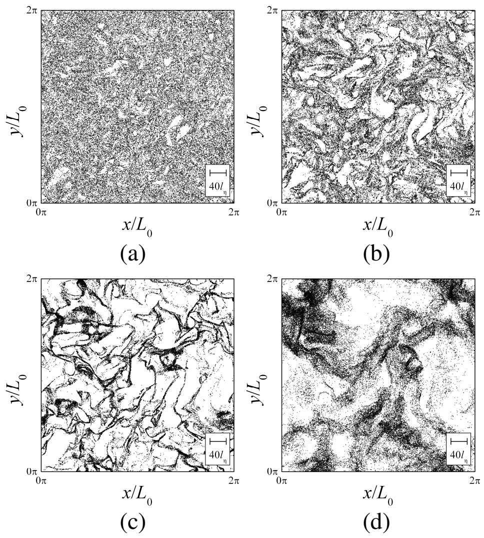

Fig. 1. Spatial distributions of droplets obtained by DNS for (a) St=0.05, (b) 0.2, (c) 1.0, (d) 5.0. Droplets located within the thin layer, whose z ranges $0<z<4l_\eta$, are drawn.

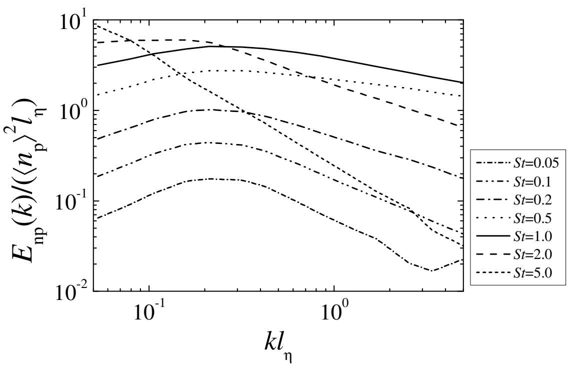

Fig. 2. Power spectra of droplet number density fluctuation obtained by DNS.

droplet number density is mainly produced by small-scale eddies. In order to evaluate Z by Eq. (2), an empirical model of $E_{np}(k)$ has been developed as a function of kl_η and St by fitting curves to the power spectra in Fig. 2.

4.2 Influence on Radar Observation of Clouds

In order to investigate the influence of turbulent clustering on a radar observation, this study evaluated the radar reflectivity factor for the high-resolution cloud-simulation data of Onishi and Takahashi [13]. They have performed a mesoscale simulation of convective clouds over the ocean by using the MSSG (Multi-Scale Simulator for the Geoenvironment) model of the Earth Simulator Center (ESC). The domain size is 12.8 × 12.8 × 4.0 km. A spectral-bin scheme is used for water droplets to take account of droplet size distributions explicitly.

In this study, the droplet size distribution in each grid cell was taken into account for calculating Z. The frequency of microwave is set to 2.8 GHz in S-band, which is often used for radar observations. Fig. 3 shows the volume rendering of clouds and radar reflectivity factor with and without the effect of turbulent clustering in the domain with 2.4 × 0.1 × 4.0 km. Comparison between Fig. 3(b) and (c) clearly shows that turbulent clustering dramatically increases the radar reflectivity factor. This result indicates that turbulent clustering may well cause a significant error in the microwave radar observations.

5 Conclusions

This study investigated the effect of turbulent clustering of droplets on radar reflectivity factor. A three-dimensional direct numerical simulation (DNS) for particle-laden isotropic turbulence has been performed to obtain clustering data for constructing a radar reflectivity factor model. The model is based on empirical fitting to the power spectrum of droplet number density fluctuation obtained from the DNS. One significant feature of the model is that it considers the Stokes number dependency. The model has been applied to a 2.8 GHz microwave emitted on the high-resolution cloud-simulation data obtained by the MSSG model. The result has revealed that the turbulent clustering significantly increases the radar reflectivity factor for the 2.8 GHz microwave. This indicates that turbulent clustering may well cause a significant error in microwave radar observations.

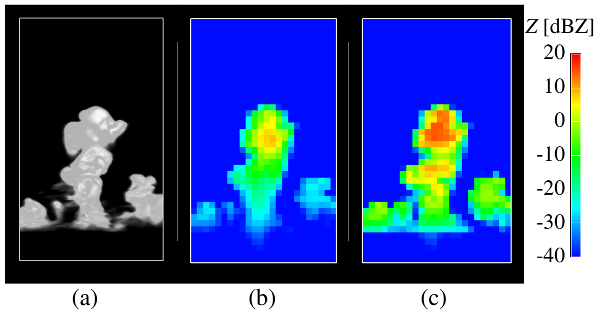

Fig. 3. Distributions of radar reflectivity factor, Z [dBZ], on x-z plane: (a) visualization of clouds; (b) Z without clustering effect; (c) Z with clustering effect.

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