Surface zonal flows induced by thermal convection in a rapidly rotating thin spherical shell

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1 Introduction

Surface flows of Jupiter and Saturn are characterized by the broad prograde jets around the equator and the narrow alternating jets in mid- and high-latitudes. It is not yet clear whether those surface jets are produced by convective motions in the "deep" region, or are the result of fluid motions in the "shallow" weather layer.

"Shallow" models, where the thickness of the atmospheric layer is quite smaller than the radius of the planet, can produce narrow alternating jets in mid- and high-latitudes, while the equatorial jets are not necessarily prograde [1]. On the other hand, "deep" models, where the thickness of the atmospheric layer is comparable to the radius of the planet, can produce equatorial prograde flows easily, while it seems to be difficult to generate alternating jets in mid- and high-latitudes [2]. Recently, Heimpel and Aurnou [3] consider a thin spherical shell model and show that the equatorial prograde jets and alternating jets in mid- and high-latitudes can produce simultaneously when the Rayleigh number is sufficiently large and convection becomes active inside the tangent cylinder. However, in that study, eight-fold symmetry in the longitudinal direction is assumed and fluid motion is calculated only in the one-eighth sector of the whole spherical shell. The artificial limitation of the computational domain may influence on the structure of the global flow field. For example, zonal flows may not develop efficiently due to the insufficient upward cascade of two-dimensional turbulence, or stability of mean zonal flows may change with the domain size in the longitudinal direction.

In the present study, we perform numerical simulations in the whole thin spherical shell domain while coarse spatial resolution and slow rotation rate compared to [3] are used due to the limit of computational resources.

2 Formulation and experiments setup

We consider Boussinesq fluid in a spherical shell rotating with angular velocity Ω. The non-dimensionalized governing equations consist of equations of continuity, motion, and temperature [4]. The non-dimensional parameters appearing in the governing equations are the Prandtl number, $\mathrm{Pr} = \nu/\kappa$, the Ekman number, $\mathrm{E} = \nu/(\Omega D^2)$, and the Rayleigh number, $\mathrm{Ra} = \alpha g_o \Delta T D^3/(\kappa\nu)$, where $\nu, D, \kappa, \alpha, r_o, g_o$, and ΔT are kinematic viscosity, the shell thickness, thermal diffusivity, the outer radius of the shell, the thermal expansion coefficient, the acceleration of gravity at the outer boundary, and the temperature contrast between the boundaries, respectively. The spherical shell geometry is defined by the radius ratio, $\chi = r_i/r_o$, where r_i is the inner radius of the shell. Both inner and outer boundaries are isothermal, impermeable and stress free. The initial conditions are zero velocity field with 4th-fold symmetry temperature perturbation used in the dynamo benchmark[4]. The control parameters used in this study are summarized in Table 1. The radius ratio χ is slightly reduced compared to the previous study [3] due to the limit of computational resources.

3 Numerical method

The numerical method used in the present study is a traditional spectral method, where nonlinear terms are evaluated with the transform method [5].

The toroidal and poloidal potentials of velocity are introduced in order to satisfy the equation of continuity. The velocity potentials and the temperature field are expanded horizontally by the spherical harmonic functions and radially by the Chebychev polynomials. The nonlinear terms of the governing equations are evaluated in the physical space and are converted back into the spectral space. The time integration is performed using the Crank-Nicolson scheme for the diffusion terms and the second-order Adams-Bashforth scheme for the other terms.

Our numerical code is parallelized using both MPI and OpenMP. The MPI implementation is quite simple. Each Processor Element (P.E.) keeps the whole spectrum data and the grid data divided in the latitudinal direction. The inverse transformation are performed in a parallel way on each P.E. After inverse transformation, the spectrum data are synchronized with all P.E.

In the present study, The total wave number of spherical harmonics is truncated at 170, and the Chebychev polynomials are calculated up to the 48th degree. We solve the governing equations in a full-spherical shell without assuming any symmetry. The numbers of grid points in the physical space are 512, 256, and 48 in the longitudinal, latitudinal, and radial directions, respectively. In order to save computational resources, we use hyperdiffusion with the same functional form as the previous studies [3, 6]: $\nu = \nu_0$ for $l \le l_0$ while $\nu = \nu_0[1 + \varepsilon(l - l_0)^2]$ for $l > l_0$, where l is total horizontal wave number. In this study, we set $l_0 = 85$ and $\varepsilon = 10^{-2}$ in all calculations. The model was run for over 4000 planetary rotations (100000 time step) and total CPU time is about 2 weeks.

Table 1. Values of the control parameters, $\chi, \mathrm{E}, \mathrm{Ra}, \mathrm{Pr}$, and the output parameters Re, Ro in the numerical experiments. The modified Rayleigh number Ra^* is the ratio of buoyancy force to the Coriolis force[2]. The Reynolds and Rossby numbers given here are based on the maximum value of mean zonal flow.

Control parameter	
$\chi = r_i/r_o$	0.75
$\mathrm{E} = \nu/\Omega D^2$	10^{-4}
$\mathrm{Pr} = \nu/\kappa$	0.1
$\mathrm{Ra} = \alpha g_o \Delta T D^3/\kappa\nu$	5×10^5, 10^6
$(\mathrm{Ra}^* = \mathrm{RaE}^2/\mathrm{Pr}$	0.05, 0.1)
Output parameter	
Re	$3.59 \times 10^2, 1.13 \times 10^3$
Ro	$3.59 \times 10^{-2}, 1.13 \times 10^{-1}$

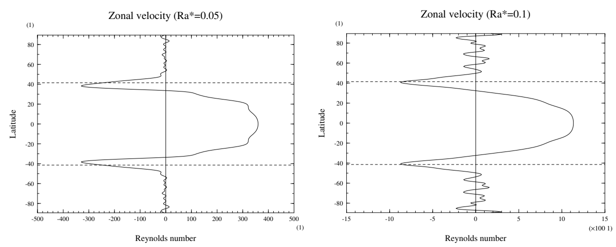

Fig. 1. Snapshot of azimuthally-averaged zonal velocity profiles at the outer surface. The left and right panels shows the cases with $\mathrm{Ra}^* = 0.05$, and 0.1, respectively. The dashed lines show the latitude at which the tangent cylinder intersects the outer surface.

4 Results

Table.1 summarizes the output results of our numerical simulations. Fig.1 shows the azimuthally-averaged zonal velocity profiles at the outer surface for two different values of the Rayleigh number. Alternating zonal jets emerge in mid- and high-latitudes when $\mathrm{Ra}^* = 0.1$ while they cannot be observed in the case of $\mathrm{Ra}^* = 0.05$. Fig.2 shows azimuthally-averaged zonal velocity distributions in a meridional cross section. It can be seen that zonal flows are almost uniform along the rotation axis. Convective motions can be observed in the tangent cylinder when $\mathrm{Ra}^* = 0.1$ while they are not active when $\mathrm{Ra}^* = 0.05$.

5 Concluding remark

Even though the spatial resolution and the rotation rate are reduced and radius ratio is slightly increased in order to solve the whole domain of the shell, we have confirmed that the equatorial prograde flows and alternating jets in mid- and high-latitudes are produced simultaneously in thermal convection in a rapidly rotating thin spherical shell. However, the results of forced two-dimensional turbulence on a rotating sphere [7] suggest that those zonal flow structure might not be maintained when time integration is further continued, which should be resolved in the future.

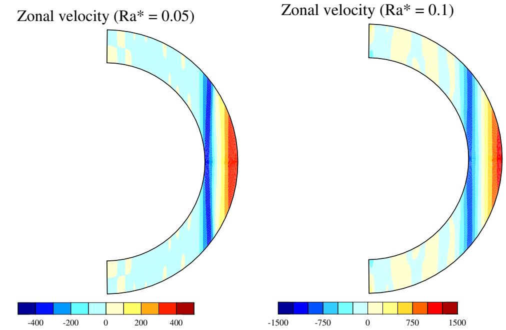

Fig. 2. Snapshot of azimuthally-averaged zonal velocity distributions in a meridional cross section. The left and right panels show the cases with $\mathrm{Ra}^* = 0.05$ and 0.1, respectively. Red and blue represent prograde (eastward) and retrograde (westward) flows, respectively.

Acknowledgments

Numerical computations were carried out on the Cray XT4 at the Center for Computational Astrophysics, CfCA, of the National Astronomical Observatory of Japan.

The library for spectral transform 'ISPACK' (`http://www.gfd-dennou.org/library/ispack/`) and its Fortran90 wrapper library 'SPMODEL library' (`http://www.gfd-dennou.org/library/spmodel/`) were used for the numerical calculations.

The products of the Dennou-Ruby project (`http://www.gfd-dennou.org/arch/ruby/`) were used to draw the figures.

The numerical codes used in the present study are also available from the 'SPMODEL library' web site: (`http://www.gfd-dennou.org/library/spmodel/`).

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