An Intermittent Control of a Double Inverted Pendulum:  
Relationship between Control Strategy and Hip Joint Stiffness

Yasuyuki Suzuki 1 and Taishin Nomura 1

1Graduate School of Engineering Science, Osaka University, Japan

Abstract
Presence of posture sway implies that human upright posture is flexibly controlled. A number of researches have been performed to clarify neural control mechanisms of the human upright standing in which a single inverted pendulum model has often been used as the human body. We have proposed a state-dependent intermittent control as the neural mechanism, and showed that the intermittent control can reproduce the flexible upright posture by the smart usage of unstable dynamics of the saddle type upright equilibrium. However, single pendulum models are not sufficient to discuss about joint coordination among multiple body segments during standing. Therefore, we constructed a double inverted pendulum model with the state-dependent intermittent control and analyzed it. In this presentation, we introduce a theoretical basis of the intermittent control, and then discuss relationship between hip joint passive stiffness and control strategy.

Keywords – Posture control, Intermittent control, Joint coordination

1 Introduction
The center of mass of human body fluctuates during quiet standing, referred to as the postural sway. This fluctuation indicates that the upright posture is controlled flexibly. The posture control consists of passive control, which arises from mechanical joint properties (e.g., viscosity and elasticity), and active control, which acts through the central nervous system with feedback time delay. The active control is necessary to keep the posture, because the quiet standing is a saddle type unstable equilibrium if active control is absent. One of the major issue in the field of postural control is whether the active control is continuous or intermittent[1-4].

We have proven that the central neural control to keep the posture is a state-dependent intermittent control[3,4]. We consider that the nervous system smartly uses the unstable dynamics of the saddle type equilibrium by switching between activation and inactivation of active torque intermittently depending on the state. We have shown that the state-dependent intermittent control can reproduce the human flexible quiet standing, which cannot be shown by the continuous control, however, the discussion has been continued.

In many cases, the human body during quiet standing is approximated by a single inverted pendulum. However, recent studies have shown that the hip joint motion is as large as or even larger than the ankle joint motion. Moreover, it is indicated that coordination between joints plays an important role to keep the quiet standing posture. For better understanding of the nervous control mechanism, it is required to analyze the quiet standing using multi-link human body model.

To this end, we consider a double inverted pendulum model during quiet standing in sagittal plane. We assume that the active torques act intermittently depending on the state of the double inverted pendulum. In this presentation, we introduce the theoretical basis of state-dependent intermittent control. And then, we discuss about joint coordination through the analyzing relationship between elasticity of joint and control strategy taken by the intermittent control to keep the quiet standing posture.

2 Double Inverted Pendulum Model
We construct a double inverted pendulum model during human quiet standing (Fig. 1). Two joints correspond to ankle and hip joints. Motion equation of the double inverted pendulum is described as follows:

$$M\ddot{\theta} + G\dot{\theta} = Q$$

where $M$ is the inertia matrix, $\theta$ the joint angle vector, $G\theta$ the gravitational toppling torque vector, and $Q$ the joint torque vector at the ankle and the hip. Implementation of $Q=(\theta_a, \theta_h)^T$ varies depending on a condition of activation or inactivation of active control at ankle joint and hip joint. We consider that activation and inactivation at each joint can be independently switched with time. Therefore, we can take the double inverted pendulum model controlled by intermittent activation at each joint as hybrid system which consists of four types of feedback control models (see Table for relationship between model name and active torque condition). During the off-off model duration, $Q$ consists of only passive torque $T_p=(\tau_{\theta_a}^p, \tau_{\theta_h}^p)^T$ ($\tau_{\theta_i}^p=-K_i\dot{\theta}_i-B_i\dot{\theta}_i$ ($i=a,h$)). During the on-model, off-on model, and on-on model duration, $Q$ is implemented as $T_p+(\tau_{\theta_a}^p, \tau_{\theta_h}^p)^T$, and $T_p+(\tau_{\theta_a}^p, \tau_{\theta_h}^p)^T$ ($\tau_{\theta_i}^a-P_i\dot{\theta}_{\theta_i}+D_i\dot{\theta}_{\theta_i}$ ($i=a,h$)), respectively. Here, $x_i$ denotes inclusion of delay, i.e. $x_i(t\rightarrow t-a)$.

Table. Base models of Hybrid system

<table>
<thead>
<tr>
<th>Model name</th>
<th>ankle joint</th>
<th>hip joint</th>
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<tbody>
<tr>
<td>off-off model</td>
<td>inactivated</td>
<td>inactivated</td>
</tr>
<tr>
<td>on-off model</td>
<td>activated</td>
<td>inactivated</td>
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<td>off-on model</td>
<td>inactivated</td>
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<td>on-on model</td>
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Fig. 1. Double inverted pendulum model during quiet standing

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2.1 Theoretical Basis

In the state-based intermittent control model, dynamics near the stable manifold of the off-off model is smartly used to maintain the standing posture. Here, we illustrate the theoretical basis of the intermittent control. When the state is located close to the stable manifold of the off-off model in the off-off model duration, it transiently approaches the equilibrium point, and then moves away from the equilibrium according to the unstable manifold of the off-off model (Fig. 2(a)). When the state is located far from the stable manifold of the off-off model, because it is not expected that the state reaches equilibrium if the off-off model is continued, the model is switched to the other model which transverse the state stable manifold of the off-off model (Fig. 2(b)). When the state transverse the stable manifold of the off-off model, the model is switched to the off-off model, and then, state approaches the equilibrium again (Fig. 2(c)). By repeating these protocols, the state-based intermittent control keeps the state bounded region near the equilibrium.

2.2 Switching Rule

As described above, the state-based intermittent control smartly uses dynamics of the stable manifold in the phase space of the off-off model. The switching rule to realize the theoretical basis consists of following three conditions:

I. When the state is located in the neighborhood of the stable manifold of the off-off model, the system is governed by the off-off model.

II. When the state is located not in the neighborhood of the stable manifold of the off-off model but that of the on-off model, the system is governed by the on-off model.

III. Otherwise, the system is governed by either the on-off, the off-on, or the on-on model, according to vector field information of each model.

3 Relationship between the hip joint elasticity and the control strategy

Figs. 3(a), (b), and (c) show sample time series data of the intermittent control model at the condition of large, medium, and small hip joint elasticity, respectively. From these dynamics, we can confirm that the control strategy depends on the value of hip joint elastic coefficient. At the condition of large hip joint elasticity (Fig. 3(a)), appropriate switching of activation and inactivation of active control at ankle joint can stabilize the equilibrium of the double inverted pendulum, even if the active control at hip joint is continuously inactivated. This means that the active control at ankle joint plays important role at the large hip joint elasticity condition. We refer to this intermittent control mode as the intermittent ankle strategy. In contrast, at the condition of medium hip joint elasticity (Fig. 3(b)), simultaneous activation of active control at ankle joint and that at hip joint is required for the stabilizaton. We refer to this intermittent control mode as the intermittent mixed strategy. The strategy, at the condition of small hip joint elasticity (Fig. 3(c)) is better to be referred to as the intermittent hip strategy, because active control of hip joint play an important role to keep the standing posture. In fact, at the small hip joint elasticity condition, activation and inactivation of active control of hip joint, i.e. switching between the off-off model and the off-on model, can stabilize the equilibrium.

As observed above, three types of strategies or synergies spontaneously emerge as a function of the elasticity of hip joint, though the switching rules between models is common through the conditions.

4 Conclusion

We constructed a double inverted pendulum model during human quiet standing posture. We illustrated a theoretical basis of the state-dependent intermittent control, and showed that the intermittent control can stabilize the equilibrium of the double inverted pendulum. Moreover, we showed that the three types of strategies to maintain the posture spontaneously emerge depending on the values of elastic coefficient at hip joint. For a better understanding of the control mechanism of human quiet standing, further experimental measurements and analysis based on these knowledge are required.

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References