Modeling and Simulation of MEMS Accelerometers and Gyroscopes Using Order-Reduction Methods

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Abstract

Current research in design methods for integrating micro electromechanical systems (MEMS) in system-on-chip (SoCs) shows that there is a need for new modeling and simulation techniques. This is because the current modeling techniques, finite element analysis, consumes an enormous amount of resources: numerical computation, time and designer's effort. We suggest implementing behavioral modeling of MEMS and applying methods of order-reduction (MOR) to these models to save time and effort in both design process and simulation. In this paper, we implemented MEMS accelerometer and gyroscope behavioral models and utilized singular values decomposition (SVD) and orthogonal-triangular decomposition (QR) methods to these models for the purpose of order reduction and rank approximation of the input system. This resulted in improving the simulation time and system resources.

Keywords − Micro electromechanical systems, accelerometer, gyroscope, Order-reduction method, SVD

1 Introduction

The integration of microelectromechanical systems (MEMS) as virtual blocks in system-on-a-chip (SoC) poses many new challenges to design and test engineers. SoCs already embed typical sub-systems such as DSP, RAM, MPEG cores, etc., and they may soon include MEMS [1]. The absence of proper MEMS design methodology for integration in SoC is a major problem; this includes the need of new modeling and co-simulation techniques.

Because of the physical nature of MEMS devices, their behavioral models consists of a huge number of mathematical partial differential equations (PDE), which consumes long time and effort for computation, processing, and simulation. However, any slight change in the design scheme, which is inevitable in the design process, leads to changes in the mathematical model, and consequently, to more time and effort loss. Order-reduction methods application to these models can result in speeding up simulation using a more reduced-order model (ROM) which represents the MEMS device behavior with small loss in precision.

Based on recent market research, MEMS accelerometers and gyroscopes constitute more than half the MEMS market. MEMS accelerometers and gyroscopes are used for micromachined inertial measurement units (IMUs), which started to appear in the market in the past decade as low cost, moderate performance alternative in many inertial applications including military, industrial, medical, and consumer applications.

In this paper, we introduce an implementation of MEMS accelerometer and gyroscope behavioral models (instead of finite element method) and apply singular values decomposition (SVD) and orthogonal-triangular decomposition (QR) to these models as order-reduction methods. Currently, SVD is being applied to image processing and compression, molecular dynamics, small angle scattering, information retrieval, electric circuit analysis, and others. In our approach, we use SVD on MEMS models, because SVD is suitable for overdetermined systems (as discussed in section 3). The advantage of our approach is that we represent the system by its behavior, thus reduce the design process time and designer's effort; moreover, we lower the order and rank of the input system, and improve the simulation time.

2 MEMS Devices Modeling

Modeling of MEMS accelerometers and gyroscopes progressed from methods which depend on finite element analysis and boundary element analysis to joint modeling, as in the case of MEMS+ tool from CoventorWare, which offers the MEMS model as a black box presentation into MATLAB for further processing, simulation, and optimization. In our case, we performed modeling of these devices depending on their physical behavior in MATLAB/Simulink. We used Simscape and SimElectronics block libraries in Simulink.

The behavior of MEMS accelerometer is represented by a mass-spring-damper system where the input is a force, following the equation: $u(t) = M.x'' + C.x' + K.x$ where M is a mass moving relative to the accelerometer, C is the damping constant, and K is the spring (stiffness) constant. The output of the MEMS accelerometer is a voltage value proportional to the acceleration caused by a force applied on a mass, with minimum output value being 1 V, output values for zero acceleration being 5 V, and maximum output value equals 10 V.

On the other hand, the behavior of MEMS gyroscope is represented by a mass-spring-damper system where the input is a force, following the equations: $u(t).sin(wt) = M.x'' + C.x' + K.x$ and $u(t) = -2.M.\Omega.v$ where w is the $resonant frequency = squareroot(K/M)$, Ω is the angular velocity, v is the translational velocity. The model output is a voltage signal ranging from 1 to 12 V with 2.5 V indicating no rotation, where this signal is proportional to the rotational velocity.

In order to simulate the above behavioral models, we

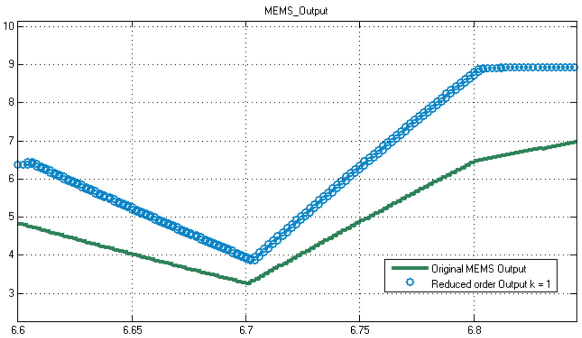

Fig. 1. MEMS Accelerometer output signals: original (green line); reduced rank = 1 (blue circles)

need to construct the input matrix $A(m \times n)$. A consists of multiple input vectors ai, where $i = 1 \ldots n$. Each vector ai consists of m input force values with constant values for C_i and K_i throughout the vector. In our system, we set the dimension of A to be 5000×10. Each a_i is constructed by the following values of C and K respectively:
$C = 0.1, 0.5, 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5$
$K = 10, 50, 100, 150, 200, 250, 300, 350, 400, 450$

In the following section, we apply an order-reduction method on matrix A in order to get a reduced-order matrix. Next, we simulate the models with both the original input matrix and the reduced one.

3 Order-Reduction Methods

We chose to apply SVD method on both models of the accelerometer and gyroscope, since SVD is suitable for solving overdetermined systems of linear equations: $A \times x = b$, as in the case of MEMS, where A is the input $m \times n$ matrix and $m > n$, i.e. with more equations than unknowns, and b is the output matrix of dimension $m \times n$. This is due to the need to repeatedly take measurements in order to minimize errors. Moreover, we checked the obtained results using QRD. For further information about these methods, we refer the reader to references [2, 3].

Based on SVD, A is decomposed into $U \times S \times V^T$. The suggested models are of type SISO (single-input single-output). In MATLAB, we use: `[U, S, V] = svd(A)` where U and V are left and right singular vectors, and S is diagonal singular values (importance coefficients), and x can be found by: $x = A^{-1} * b$, where $A^{-1} = V \times S^{-1} \times U^T$ Also, QR was applied: `[Q, R] = qr(A)` where $A = Q \times R$, and $x = Q^T \times R^{-1} \times b$. In every simulation, the values of x were checked from both methods and they were always equal. Matrix A can be reconstructed with the reduced rank $k < n$ as follows: $A_k = U(:, 1 : k) * S(1 : k, 1 : k) * Vt(:, 1 : k)$ and the reduced-order matrix can be considered as the projection of matrix A into the subspace V, i.e. $A_{red} = A * V(1 : k, :)$.

4 Simulation

Both models were simulated for 100 seconds to generate arbitrary input signals, after which the input matrix A is constructed. After implementing SVD, the same model is simulated but this time with each reduced A matrix of rank $k < n$, i.e. $k = 1 \ldots n - 1$. In our system, simulation was performed with input set: $n = 10$, $m = 5000$.

We define the error as the difference between the outputs

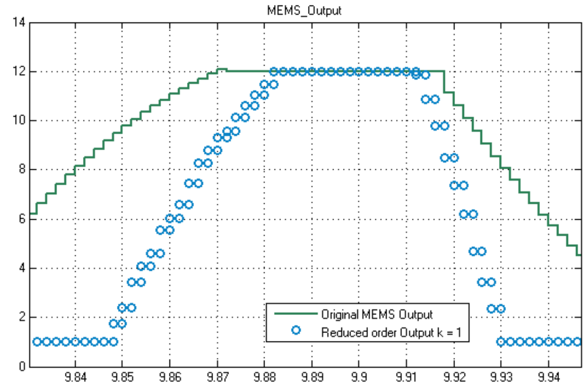

Fig. 2. MEMS Gyroscope output signals: original (green line); reduced rank = 1 (blue circles)

Table 1. Simulation times for orders $k = 1 \ldots 10$, for MEMS accelerometer and gyroscope

Order k	1	2	3	4	5	6	7	8	9	10
Accel.	8	8	11	16	20	23	27	31	34	97
Gyro.	2	2	2	3	3	3	3	3	3	19

of the reduced model b_k of order k and the original model b_o of order n, divided by output of the original one: $er = abs(b_k - b_o)/b_o \times 100$.

For MEMS accelerometer, average errors for $k = 1 \ldots 10$ are approximately equal to 0.1863. In Fig. 1 we can notice that the output signal generated from A of order 1 has an approximate form of the original output. Also, the diagonal singular values of matrix S are $s1 = 11130, s_2 = 1044.1, etc.$. Since $s_2 \ll s_1$ so it can be neglected, and the new rank of A will be 1. So the optimal reduced order of A, $k = 1$ verified graphically and through the importance coefficient. The same thing is noticed in Fig. 1 with the MEMS gyroscope, where average errors for $k = 1 \ldots 10$ are approximately equal to 0.0652.

5 Conclusion

In this paper, MEMS accelerometer and gyroscope were modeled in Simulink. In order to improve simulation, and decrease simulation time, SVD and QR methods were applied. Also, optimal reduced orders were defined for the input sets used in Fig. 1 and 2 and through the importance coefficients of matrix S.

In Table 1 Simulation time for MEMS accelerometer was reduced by 88% of the simulation time of the original system, i.e. the new simulation time is 12% of the original system. As for MEMS gyroscope, the simulation time was reduced by 90.5% of the simulation time of the original system, i.e. the new simulation time is 9.5% of the original system. As for memory resources, decrease of memory usage was by 90%, from 400,000 bytes to 40,000 bytes.

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