Mesh Coarsening Method Based on Multi-point Constraints for Large Deformation Finite Element Analysis of Almost Incompressible Hyperelastic Material

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Abstract

In this study, we propose a mesh coarsening method based on the multi-point constraints for large deformation finite element analysis. In this coarsening method, a distorted element and the neighbor quality elements is combined into one element. We analyze large deformation of mixture of almost incompressible Mooney-Rivlin material using the u/p formulation in the total Lagrangian description and St. Venant-Kirchhoff material with a large Young’s modulus. In a finite element analysis of such materials, mesh distortion arises near the material interface because of the difference of Young’s modulus and it leads to termination of a finite element analysis program. From the numerical results, it is confirmed that the mesh coarsening method is effective for mesh distortion in the vicinity of singularity. With the present method, it is possible to improve the Jacobian values of elements around the coarsened elements and continue further computation.

Keywords - Mesh Coarsening, Multi-Point Constraints, Large Deformation, Material Interface, Hyperelastic Material

1. Introduction

Rubber material analysis is widely utilized in fields such as automotive engineering, civil engineering, and bioengineering. One of the important research themes is small-scale analysis of rubber compound including filler, which is often used in tire industry. Additional fillers can increase hardness, strength, and wear resistance of rubber products. In this paper, rubber and filler are modelled as almost incompressible hyperelastic material and St. Venant-Kirchhoff material respectively. We analyz large deformations of almost incompressible hyperelastic material using the u/p finite element formulation in the total Lagrangian description [1]. While the Young’s modulus of filler is the order of 10⁶ Pa, that of rubber is on the order of 10⁷ Pa or 10⁸ Pa. In a finite element analysis of such materials, mesh distortion arises near the material interface because of the difference of Young’s modulus and the Jacobian of an extremely distorted element becomes negative. Even though the negative Jacobian arises only near the interface, it leads to termination of a finite element analysis program.

In many researches on finite element analysis, the use of a high-quality finite mesh is frequently effective in order to obtain accurate numerical solutions for large deformation problems. For example, an arbitrary Lagrangian-Eulerian (ALE) finite element formulation by Yamada and Kikuchi 1993 has been studied so far [2]. However, in order to obtain acceptable numerical solutions, the use of a high-quality finite mesh is not necessarily effective in the vicinity of singularity and it might cause increases of singularity and further mesh distortion. It is shown that application of a comparatively finer ALE mesh is not successful for mesh distortion in the vicinity of singularity in Yamada and Kikuchi 1993.

In this study, we propose a mesh coarsening method based on the multi-point constraints (MPC) [3]. In this coarsening method, a distorted element and the neighbor quality elements is combined into one element by the MPC. The approach is based on the idea that application of coarser mesh is effective for mesh distortion in the vicinity of singularity to obtain acceptable numerical solutions.

2. Mesh Coarsening Method Based on MPC

An analysis mesh is deformed as shown in Fig. 1 (left figure). Element (e₁) near the material interface is distorted and the Jacobian of Element (e₁) becomes negative soon. Finally the computation is terminated. In the reference configuration, we combine Element (e₁) with the neighbor high quality element, Element (e₂), which has the largest Jacobian. The connectivity between Node C and Node F are deleted and a new element, Element (e₃), is created as shown in Fig. 1 (right figure). In the new analysis mesh, nodal displacements \( u_C^t \) and \( u_F^t \) are computed by the MPC as follows:

\[
\begin{align*}
\frac{1}{2} u_C^t &- \frac{1}{2} u_X^t - \frac{1}{2} u_B^t = 0 \\
\frac{1}{2} u_F^t &- \frac{1}{2} u_D^t - \frac{1}{2} u_E^t = 0
\end{align*}
\]

We can compute the nodal displacements \( u_C^t \) and \( u_F^t \) of Element (e₃) without changing the connectivity of surrounding elements. After the mesh coarsening, we start computing from the reference configuration.

3. Numerical Examples

We deal with simple deformation of a mixture of the Mooney-Rivlin material and the St. Venant-Kirchhoff material. The initial geometrical shape of an analysis model is shown in Fig. 2. The analysis model is composed of Body (a) and Body (b); Body (a) is a 50mm cube which has a 20mm x 20mm square hole and Body (b) is a 20mm x 20mm x 100mm square prism.
Body (a) is an almost incompressible Mooney-Rivlin material and Body (b) is St. Venant-Kirchhoff material. As a boundary condition, the displacement at the bottom of Body (a) are fixed. The traction (nominal stress) 2.5×10^5 kPa is prescribed at the top of Body (a) in the vertical direction. The total number of loading steps for the traction is 25. The total numbers of elements of Body (a) and Body (b) are 105,000 and 40,000 respectively. We decompose the analysis mesh into 16 domains and calculate with 16 CPUs parallel computing by MPI.

In order to confirm the effect of the mesh coarsening method to mesh distortion near the material interface, we compare two conditions: Case 1 and Case 2. In Case 1, we compute without the mesh coarsening method. The negative Jacobians arise for finite elements of Body (a) and the computation terminates at the loading step 23. In Case 2, we apply the mesh coarsening method only to distorted elements with the material interface at the loading step 22 in Case 1, and compute once more from the loading step 22. The number of distorted elements with the material interface is 200 at the loading step 22 in Case 1.

It is possible to continue computation with mesh coarsening until the final loading step 25. The deformation of finite elements in the computation with mesh coarsening is shown in the left of Fig. 3, and the right of Fig. 3 shows the elements which have the negative Jacobians near the material interface. In Case 2, the number of elements which have the negative Jacobian decreases mainly near the lower corner of Body (b). Figure 4 shows the deformation of finite elements near the material interface at the loading step 22. In Fig. 4, Element (e_1) has the negative Jacobian, and Element (e_2) is created from Element (e_1) and Element (e_3) by applying the mesh coarsening method. It seems that finite elements near the material interface in Case 1 are more distorted than in Case 2. There is no large difference between the Jacobian of Element (e_1) and that of Element (e_3). However, Element (e_1) in Case 2 improves the quality of elements which have negative Jacobian around Element (e_2). For example, the minimum values of nodal Jacobians of Element (e_2) in Case 1 and Case 2 are -6.02×10^-3 and 1.04×10^-1 respectively; those of Element (e_3) in Case 1 and Case 2 are -8.57×10^-4 and 1.18×10^-2 respectively. Figure 5 shows the total number of elements which have the negative Jacobian at each loading step in both Case 1 and Case 2. It is confirmed that the mesh coarsening approach proposed in this paper sufficiently suppresses increase of the total number of elements which have the negative Jacobians.

4. Conclusion

In this study, we proposed the mesh coarsening method based on the MPC. As the verification, we analyzed the simple tensile deformation of a mixture of the Mooney-Rivlin material and the St. Venant-Kirchhoff material with 16 CPUs parallel computing by MPI. While the computation was terminated at the loading step 23 without mesh coarsening, it is possible to compute until the final loading step 25 with mesh coarsening. With the mesh coarsening method, it is possible to improve the Jacobian values of elements around a coarsened element. It was confirmed that the mesh coarsening approach proposed in this paper sufficiently suppresses increase of the total number of elements which have the negative Jacobians.

The fundamental idea of this approach is available to obtain acceptable numerical solutions. We plan to address more practical problems where rubber compound includes multiple fillers, for which it is necessary to consider a threshold of the Jacobian for deleting the connectivity of elements.

References