An Emergency Evacuation Planning Model Based on the Universally Quickest Flow

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Abstract

In this article, we formalize the emergency evacuation planning model for evacuation from tsunamis and other disasters based on the idea of the universally quickest flow. We show that there does not always exist a universally quickest flow when the capacity constraint of refuges is taken into account. Therefore, we propose an alternative criterion that approximates a universally quickest flow, and presents an algorithm for finding an optimal flow under this criterion. Numerical experiments are carried out for the coastal area of Tokushima City in Japan.

Keywords - Emergency evacuation planning model, Universally quickest flow, Tsunami evacuation building

1 Introduction

In recent years, catastrophic disasters by massive earthquakes have been increasing in the world, and disaster management is required more than ever. For example, in the Tohoku-Pacific Ocean Earthquake that happened in Japan on March 11, 2011, serious damages were caused by a tsunami. Although disaster prevention in Japan has been considered previously mainly from physical aspects, it turns out that it is difficult to prevent large tsunamis physically. Therefore, disaster prevention from city planning and evacuation planning has become more important.

The authors have formalized the evacuation planning problem by using the quickest flow model that minimizes the evacuation completion time of evacuees [2]. Let us denote this time by Θ^*. This criterion focuses on the time to complete evacuation of the last evacuee. However, in the case of disasters such as tsunamis in which a slight delay in evacuation may deprive evacuees' life, minimizing $\Theta^*$is not sufficient, but we need to take into account the other criterion that maximizes the number of evacuees who have already been evacuated at an arbitrary time before Θ^*. The problem with this criterion is called the universally quickest flow [3]. When we apply this model to the emergency evacuation problem, we have to consider the capacity constraints of refuges. In this situation, we will show a counterexample in which there does not exist a universally quickest flow. We then propose an alternative criterion, which approximates a universally quickest flow, and then we present an algorithm for finding an optimal flow with this criterion. Finally, numerical experiments of evacuation planning are carried out for Tokushima city in Japan where tsunami damages are predicted when the Nankai earthquake occurs.

In contrast with evacuation simulation, the usefulness to use the above mathematical approach is that it gives us a mathematically guaranteed lower bound of the time needed for all evacuees to complete evacuation. This gives us an evidence for evacuation planning [1].

2 Preliminaries

We denote by $\mathbb{R}_+$ and $\mathbb{Z}_+$ the sets of nonnegative reals and nonnegative integers, respectively. Before explaining our algorithm, we explain the basis of related network flow models.

2.1 Dynamic networks

Let $D = (V, A)$ be a directed graph. Let $N = (D = (V, A), c, \tau, b, S)$ be a dynamic network which consists of a directed graph D, a capacity function $c: A \to \mathbb{R}_+$, a transit time function $\tau: A \to \mathbb{Z}_+$, a supply function $b: V \to \mathbb{R}_+$ and a set of sinks $S \subseteq V$. Since we consider the evacuation to $s \in S$, we assume without loss of generality that $b(s) = 0$ for any $s \in S$.

We define a *dynamic flow* $f: A \times \mathbb{Z}_+ \to \mathbb{R}_+$ as follows. For each $e \in A$ and $\theta \subseteq \mathbb{Z}_+$, we denote by $f(e, \theta)$ the flow rate entering e at time step θ which arrives at $h(t)$ at the time step $\theta + \tau(e)$. We call f feasible if it satisfies the following three types of constraints:

(a) The capacity constraint :

$$0 \le f(e, \theta) \le c(e) \quad \text{for any e} \in \text{A and } \theta \in Z_+. \qquad (1)$$

(b) The flow conservation constraint:

$$\sum_{e \in \delta_D(v)} \sum_{\theta=0}^{\Theta} f(e, \theta) - \sum_{e \in \rho_D(v)} \sum_{\theta=0}^{\Theta-\tau(e)} f(e, \theta) \le b(v) \qquad (2)$$

for $\forall v \in V$ and $\forall \Theta \in \mathbb{Z}_+$.

(c) The demand constraint: $\exists \Theta \in \mathbb{Z}_+$ such that

$$\sum_{s \in S} \sum_{e \in \rho_D(s)} \sum_{\theta=0}^{\Theta-\tau(e)} f(e, \theta) = b(V). \qquad (3)$$

For a feasible dynamic flow f, we define the evacuation time of f by the minimum Θ^* satisfying (3) and call it the quickest flow.

2.2 Time-expanded network

Let $N = (D = (V, A), c, b, S)$ be a static network which consists of a directed graph $D = (V, A)$, a capacity function $c: A \longrightarrow \mathbb{R}_+$, a supply function $b: V \longrightarrow \mathbb{R}_+$ and a set of sinks $S \subseteq V$. We call $f: A \longrightarrow \mathbb{R}_+$ a feasible flow if it satisfies the capacity constraint

$$0 \le f(e) \le c(e) \ \ (\forall e \in A), \qquad (4)$$

and the flow conservation

$$\sum_{e \in \delta_D(v)} f(e) - \sum_{e \in \rho_D(v)} f(e) = b(v) \ \ (\forall v \in V \setminus S). \qquad (5)$$

To solve the decision version of the evacuation problem with time horizon Θ, Ford and Fulkerson [4] introduced the *time-expanded network* $N(\Theta)$, which is a static network in which for each $v \in V$ and $\theta \in \{0, \ldots, \Theta\}$ there exists a node $v(\theta)$, and for each $e = uv \in A$ and $\theta \in \{0, \ldots, \Theta - \tau(e)\}$ there exists an arc $e(\theta) = u(\theta)v(\theta + \tau(e))$ whose capacity is $c(e)$, and for each $v \in V$ and $\theta \in \{0, \ldots, \Theta - 1\}$ there exists a holdover arc $v(\theta)v(\theta + 1)$ with infinite capacity. For each $v \in V$, the supply of $v(0)$ is set to $b(v)$ and the supplies of all the other nodes $v(\theta)$ $(\theta \in \{0, \ldots, \Theta\})$ are set to zero. $\{s(\theta) | s \in S\}$ is aggregated into a super sink node $s^*(\theta)$ as well as an arc $(s(\theta), s^*(\theta))$ with infinite capacity (see Figure 1).

It is known [4] that there exists a feasible dynamic flow f whose evacuation completion time is at most Θ if and only if there exists a feasible flow in $N(\Theta)$. Thus, based on the binary search, the quickest flow can be found by testing the feasibility of $N(\Theta)$ for $O(\log\Theta^*)$ different Θ's.

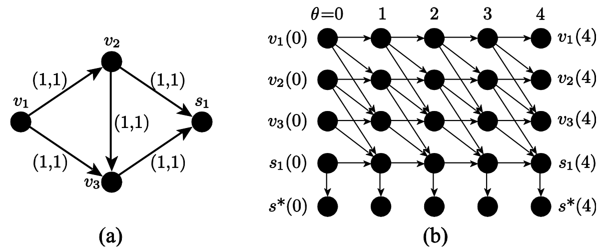

Figure 1. Illustration of a dynamic network (a) and its time expanded-network (b).

2.3 The universally quickest flow

A dynamic flow f^* which satisfies

$$\sum_{s\in S}\sum_{e\in\rho_D(s)}\sum_{\theta=0}^{\Theta-\tau(e)} f^*(e,\theta) \geq \sum_{s\in S}\sum_{e\in\rho_D(s)}\sum_{\theta=0}^{\Theta-\tau(e)} f(e,\theta) \qquad (6)$$

for $\forall\Theta$ is called the universally quickest flow.

The time-expanded network $N(\Theta)$ can be extended in order to be adapted to find the universally quickest flow. As shown in Figure 2, super sink $s^*(\theta)$ is added at each $\theta \in \{0, \ldots, \Theta\}$ and connected to each sink $s(\theta)(s \in S)$. Here, the lexicographic maximum flow considering the ordered set of sinks $\{s^*(0), \ldots, s^*(\theta), \ldots, s^*(\Theta)\}$ is the universally quickest flow when the evacuation completion time is Θ.

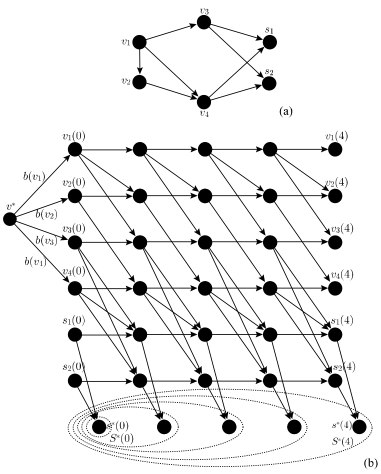

Figure 2. Illustration of a dynamic flow (a) and its time-expanded network (b) where the transit time of all arcs is 1.

Although the details are omitted, there does not always exist a universally quickest flow in the presence of the capacity constraints of refugees, we will compute instead a lexicographically quickest flow which can be obtained greedily by computing the maximum flow with a single super sink $s^*(\theta)$ at every time step θ in the order of $\theta = 1,2,\ldots,\Theta^*$. Since this flow differs from the universally quickest flow, we call it the lexicographically quickest flow (lexico-quickest flow).

3 Case study

The proposed model was applied to the Okisu area of Tokushima City in Japan. The target area is shown in Figure 3. The road network has 860 nodes and 1,106 arcs, and the population is 9,810 in this area. The number of evacuees is estimated as 7,445. The capacity of an arc is determined to be one of 2, 4 and 6 meters according to the width of the road. There are 11 tsunami evacuation buildings numbered from 1 to 11 in Figure 3. Other refuges numbered from 12 to 14 represent gateways to the outside of the Okisu area.

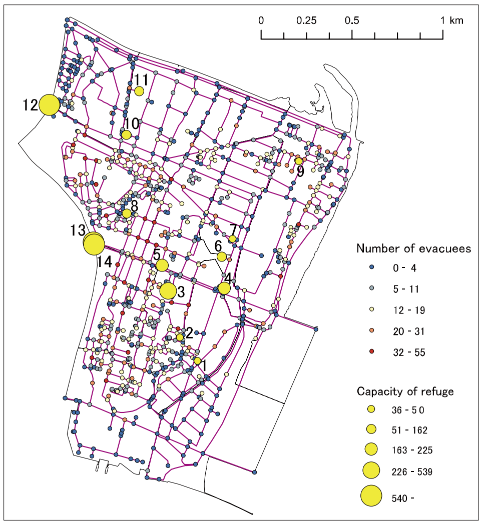

Figure 3. Detailed map of the target area

Figure 4 shows the cumulative number of evacuees for two models: the quickest flow (QC) and the lexicographic quickest flow (LC). We can see that the cumulative number of evacuees increases more rapidly at an early stage in the case of LC compared with QC. Notice that the evacuation completion time of LC is longer than that of QC by about 150 seconds.

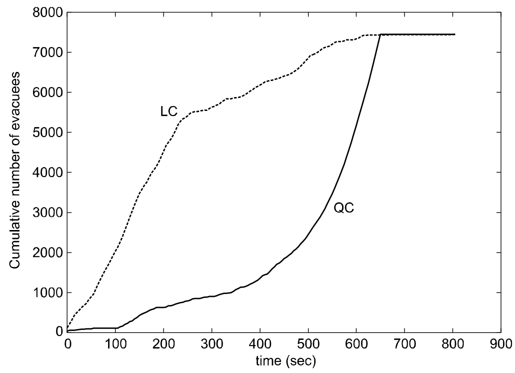

Figure 4. Comparison of cumulative numbers of evacuees for two models QC and LC.

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