Simulation Based Design for Vehicle Suspension with Inerter

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Abstract
This paper discusses an application of inerter's mechanical elements to building passive suspension systems for improving body displacement as a ride comfort. In this study, we proposed a new design, which has independent suspension, by using the inerter mechanism system allow changing the sprung and un sprung masses, and thus improving the vehicle stability with displacement on rough ground. First, we performed modeling and simulation of suspension systems with inerter, for comparing those systems with traditional suspension about a vehicle stability with displacement. And the suspension systems with best mechanism of inerter was obtained through these simulations. By using approximate optimization method with Response Surface Method, this obtained suspension system was brought close to mathematical model which was modeled ideal dynamics of its system. These results were described and discussed.

Keywords – inerter, suspension, dynamics, IDCAE, optimization, vehicle

1 Introduction
Suspension systems have been conflicted between ride comfort, specifically ground contact, and driveability [1]. Therefore, passive, semi-active and active suspension systems have been applied to improve ride comfort of vehicles and their effectiveness has also been demonstrated [2]. On the other hand, for the stock-car targeted the emergent countries, desired suspension systems are not complex suspension systems, the likes of semi-active or active suspension systems, but simple suspension systems using passive mechanisms. However, in passive suspension systems, it is not easy to improve ride comfort and dynamics stability.

In this study, we introduced new suspension element, was called the inerter, into a passive suspension system for improving body displacement as a ride comfort, and performed its modelling and simulation. Obtained simulation results were described through some discussions.

2 Inerter
The inerter is presented by M.C., Smith [3]. It was defined to be a mechanical two-terminal, one-port element with the property that the equal and opposite force applied at the nodes is proportional to the relative acceleration between the nodes [3] through a rack & pinion mechanism that consists of a combination of a rack & pinion with gears (see Fig. 1).

To approximately model the dynamics of the element of Fig. 1, \( r_1, r_2, r_3, γ \) and \( m \) were defined the radius of the rack pinion, the radius of the gear wheel, the radius of the flywheel pinion, the radius of gyration of the flywheel, and the mass of the flywheel, respectively. And the following relation holds

\[
F = b(\ddot{v}_2 - \ddot{v}_1). \tag{1}
\]

The constant of proportionality \( b \) is called the inerterance and has units of kilograms. Inerter of rack & pinion mechanism was defined

\[
b = ma_2^2a_1^2, \tag{2}
\]

where \( a_1 = γ/r_3 \) and \( a_2 = r_2/r_1 \). Moreover, Inerter of ball-screw mechanism was defined [4] as follows.

\[
b = I \left( \frac{2a}{π} \right)^2, \tag{3}
\]

where \( I \) and \( P \) denote moment of inertia and pitch in ball-screw. Inerter transforms the displacement energy of suspension to kinetic energy to rotate flywheel, and stores energy \( E \)

\[
E = \frac{1}{2}b(\ddot{v}_2 - \ddot{v}_1)^2. \tag{4}
\]

3 The quarter-car model
We begin with the quarter-car model was modelled applying new parallel structure involving inerter as shown in Fig. 2. To reduce the amplitude of the maximum displacement (\( Z_2 \)), its quarter-car model is simulated because a ride comfort is proportional to the amplitude of the maximum displacement.

![Fig. 2 the quarter-car model by mathematical modelling](image)

We modelled two mathematical models for traditional spring-damper suspension and new spring-damper-inerter suspension...
(see Fig. 2). And, two computational models of 1DCAE were modelled for including effects of mechanical elements, i.e., mechanical loss in the suspension, as shown Fig. 3. Model A and B denote traditional suspension and suspension with inerter using mathematical formulation. Model C and D denote ball-screw inerter suspension and rack & pinion inerter suspension using 1DCAE, respectively. The mechanical parameters of model C and D were drawn on from the inertance value, 20kg in Table 1. And other parameters of this specification were referred the Shibaura Formula-SAE Car. As for the input displacement of this simulation, the bump road profile was used, and its profile is a stepped wave that continues for 0.5 (sec) in height 0.03 (m).

4 Simulation for spring-damper-inerter suspension

4.1 1DCAE

Displacement ($Z_d$) results of four models were shown in Fig. 4. From these results, we checked amplitude of maximum displacement of body mass $M$. All inerter models were effective to reduce amplitude of maximum displacement through comparing to the Model A. As for computational model which included mechanical effects, maximum displacement of the Model C was smaller than the Model D. We think this cause is that the mechanical loss and the moment of inertia of the Model D are larger than the Model C because the Model D has four rotational and mechanical elements—three gears and flywheel. Next, we verified whether the result of the Model B as a mathematical model and the Model C as a computational model is the same. However, the model with the smallest maximum displacement was the Model B, and it was not the same result.

4.2 Optimization considering mechanical effects

In this section, to bring the maximum displacement of Model C close to Model B, attributes $L$, $I$ and $k$ of the mechanical element in inerter is optimized. However, the maximum displacement of Model C did not improve. Therefore, attributes $L$, $I$ and $k$ of the mechanical element in overall suspension system was optimized. The Sequential Quadratic Programming (SQP) with Response Surface Method (RSM) was introduced. And, RSM was made using the Orthogonal Arrays $L_9(3^4)$ to optimize design variables $L$, $I$ and $k$ of the suspension system with Ball-Screw type’s inerter. The optimized parameters, $I = 3.0$ (kgm$^2$), $L = 0.003$ (m) and $k = 20,500$ (N/m) was obtained, and the maximum displacement was shown in Table 2. As the optimized result, we confirmed that the maximum displacement of the optimized Model C was close to the Model B, and the optimized Model C could reduce the maximum displacement smaller than Model A about 10%.

5 Conclusion

This paper has proposed suspension system applied inerter can control amplitude of maximum displacement of sprung mass. In addition, the Ball-Screw model is more effective to absorb vibration, and the result of optimized the Ball-Screw model shows that the dynamic’s behaviour of this model was close to the inerter model by mathematical formulation. Moreover, we are modelling the detailed suspension model of Ball-Screw type into a multi-wheel vehicle model to advance simulation.

References

[1] KYB, The vehicle suspension, Sankaido, 2005