Application of an improved control variate technique to a local neoclassical transport code based on the δf Monte Carlo method

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1 Introduction

Transport phenomena in a magnetically confined plasma arises from two different mechanisms: particle collision and turbulence. The former which is usually called neoclassical (NC) transport determines an irreducible minimum transport level in a plasma. It also has a close relationship to the radial electric field, the bootstrap current, and so on which affects the transport properties in a plasma through various mechanisms. Hence, the accurate evaluation of the NC transport is a key issue to investigate the confinement performance of a device although the turbulence-driven transport often surpasses the NC transport.

The drift kinetic equation which involves the guiding center motion of particles is a basis to investigate the NC transport [1] and the particle simulation is widely used to numerically solve it. Although it has a lot of advantages such as its easiness to parallelize, applicability to multi-dimensional problems, *etc.*, there is an inherent drawback of a numerical noise proportional to $1/\sqrt{N}$, where N denotes the number of markers used in the simulation. The so-called δf Monte Carlo approach only solves the deviation of the total distribution function from the time-independent part (usually Maxwellian) to reduce the noise. This δf method can be interpreted as a control variate technique for the variance reduction in the Monte Carlo framework [2].

We have developed a numerical transport simulation code, FORTEC-3D [3, 4], based on the two-weight δf Monte Carlo method [5, 6]. FORTEC-3D have an advantage of including the finite orbit width (FOW) effect in the drift kinetic equation, which has been usually neglected in conventional numerical NC transport studies but becomes important in high temperature plasmas due to the large radial drift of trapped particles and the low collisionality. It should be emphasized that the FOW effect is expected to become important in fusion reactors in the future. The FOW effect arises from the finite excursion of the guiding center orbits across the magnetic field lines and brings higher order correction to the NC transport. The FOW effect on the NC transport and the formation of the $E_{\rm r}$ has been confirmed by comparing results of FORTEC-3D and those of other codes which *does not* include it [4, 7]. The codes used for the comparison above, however, are based on the different approaches to calculate the NC transport, thus more detailed and direct investigation for the FOW effect is required to quantitatively and individually assess their influence on the NC transport. A new *local* NC transport code based on the same physical model as FORTEC-3D except for the FOW effect needs to be developed.

On the other hand, since such a local code is less time/CPU-consuming as compared to the original FORTEC-3D due to its absence of the radial motion, it can provide us to test a new improved control variate (CV) technique proposed by Kleiber, *et al.* in Ref. [8]. Their improved control variate method has been proved to reduce the numerical noise drastically when applied to a simple one-dimensional collisional model. Thus, to test the effect of the new scheme at first, the new local code, which solves the drift kinetic equation locally in four-dimensional phase space, has been developed.

In this paper, we present initial results obtained by our new local neoclassical transport code with the CV method. The rest of the paper is organized as follows: First, the two-weight δf Monte Carlo method with the new CV method is briefly reviewed in Sec. 2. Section 3 devotes to numerical results with and without the new control variate method. Finally, a summary is given in Sec. 4.

2 δf Monte Carlo method

FORTEC-3D solves the drift kinetic equation for the deviation of $\delta f(\boldsymbol{z}, t) = f - f_{\rm M}$:

$$\begin{aligned} \frac{d\delta f}{\partial t} &\equiv \left[\frac{\partial}{\partial t} + \left(\boldsymbol{v}_\parallel + \boldsymbol{v}_{\rm d}\right)\cdot\nabla - C_{\rm TP}\right]\delta f \\ &= -\boldsymbol{v}_{\rm d}\cdot\nabla f_{\rm M}, \end{aligned} \tag{1}$$

where $\boldsymbol{z}$ denotes arbitrary phase space variables, f and $f_{\rm M}$ are the total distribution function and the Maxwellian $f_{\rm M}$, $\boldsymbol{v}_\parallel = v_\parallel \boldsymbol{b}$ and $\boldsymbol{v}_{\rm d}$ denote the parallel and drift velocity of the guiding center particle, respectively, $\boldsymbol{b}$ is the normalized vector parallel to the magnetic field,and $C_{\rm TP}$ is the pitch angle collision operator. For simplicity the new local code at present only involves the pitch angle scattering and neglects the energy scattering for markers. In the Monte Carlo method, it is required to solve the eq. (1) along the trajectories of marker particles defined by $\dot{\boldsymbol{z}}$, where dot denotes the total time derivative, where the phase space coordinates, $(\psi, \theta, \zeta, v_\parallel, v_\perp)$, are used for the orbit calculation, and ψ, θ, ζ represent the radial label, poloidal and toroidal angle variables respectively, and $v_\perp = \sqrt{v^2 - v_\parallel^2}$ and v is the magnitude of the velocity of the marker. It is noted that ψ can be assumed constant in the local approach here due to the absence of the radial drift. For the two-weight δf scheme in FORTEC-3D, each marker has two weights of w_1 and w_2 and the discretized marker particle distribution function is given in the extended phase space $F = F(\boldsymbol{z}, w_1, w_2; t)$. Then the solution of eq. (1), δf and the Maxwellian $f_{\rm M}$ at any time can be evaluated

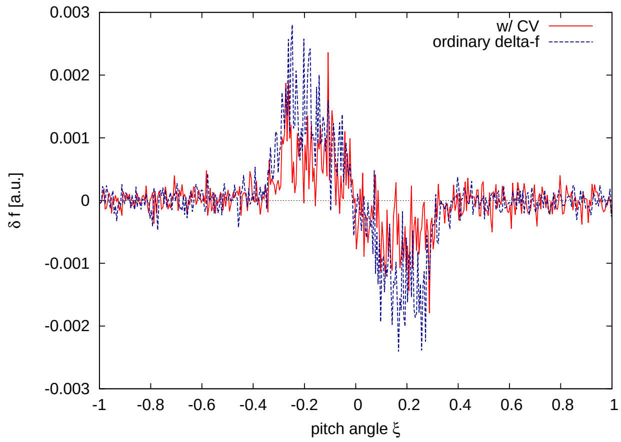

Fig. 1. The distribution function as a function of the pitch angle variable, ξ. A red solid line represents a result obtained by using the improved control variate, and a blue dashed line by the ordinary δf Monte Carlo approach.

using F as: $\delta F = \int w_1 F dw_1 dw_2$ and $f_{\rm M} = \int w_2 F dw_1 dw_2$. The time evolutions of $w_{1,p}$ and $p_{2,p}$ are given as follows:

$$\dot{w}_{1,p} = -\frac{w_{2,p}}{f_{\rm M}}\dot{\psi}\frac{\partial f_{\rm M}}{\partial \psi}, \tag{2}$$

$$\dot{w}_{2,p} = \frac{w_{2,p}}{f_{\rm M}}\dot{\psi}\frac{\partial f_{\rm M}}{\partial \psi}, \tag{3}$$

where the p subscript denotes the index of the maker particle. It is noted that the $\dot{v} = 0$ is used because the potential energy is assumed to be constant on a flux surface.

It is pointed out that the two-weight δf Monte Carlo method described above does not fully exploit the advantage of the δf procedures [8]. Further reduction of the noise is realized by introducing a new weight $\tilde{w}_{1,p} = c_p - \alpha w_{2,p}$, where $c_p = w_{1,p} + w_{2,p}$ and α is a numerical factor specified later. With the new weight, the total distribution f can be obtained by $f = \int \tilde{w}_1 F dw_1 dw_2$. The parameter α is determined to minimize the variance of the resultant f by choosing

$$\alpha \equiv \frac{{\rm Cov}\,[c, w_2]}{{\rm V}\,[w_2]}, \tag{4}$$

where ${\rm Cov}[\cdot,\cdot]$ and ${\rm V}[\cdot]$ denote the covariance and the variance of given quantities. It is noted that $\alpha = 1$ corresponds to the ordinary δf Monte Carlo method and $\alpha = 0$ to the full-f method, respectively.

3 Numerical results

The new local code has been applied to a plasma of electron species with an axisymmetric (ζ-independent) magnetic field of which the magnitude is given by, $B/B_0 = 1 - \epsilon_{\rm t}\cos\theta$, where B_0 correspond to the magnetic field strength at the core and $\epsilon_{\rm t}$ denotes the toroidicity. Here, $B_0 = 1.88$T and $\epsilon_{\rm t} = 0.05$ are used. The number of marker particles used is $32,000$.

To see how the improved control variate scheme reduces the numerical noise, the δf part of the distribution is investigated. Figure 1 shows resultant δf at the steady state as a function of the pitch angle $\xi = v_\parallel/v$. It is noted that α is evaluated only in the ξ direction, that is, the weights w_1 and w_2 are integrated in the θ and v directions to evaluate the eq. (4). As can be seen in this figure, the result obtained by the improved control variate method *does not* reduce the noise compared to that obtained by the ordinary δf Monte Carlo method. This is accounted for by the small variance of w_2 in the simulation. Since the noise in the δf method mainly arises from increase in the variance of w_2 and a resultant weight-spreading in w_1, the noise in the ordinary δf method consequently does not become large if the variance of w_2 is does not increase. In fact, the α_i factor for the improved method is nearly kept unity at all over the ξ-space, and $\alpha \simeq 1$ represent the situation that the ordinary δf method works well sufficiently.

4 Summary and discussion

In this paper, we report the application of an improved control variate technique for a local neoclassical transport code, which aims a direct comparison with our FORTEC-3D code to investigate the FOW effect in more detail. The effect of the variance reduction has been explored by the introduction of the improved control variate to the ordinary two-weight δf Monte Carlo method. The improved control variate is calculated only in the ξ-space although the drift kinetic equation solved in the code is four-dimensional. The result has shown that no significant improvement has been observed when compared to the ordinary δf method. This has occurred from the fact that the ordinary δf method has already worked well due to the small variance of the w_2.

The variance of w_2 in general increases as the radial drift of particles increases since the evolution equation is proportional to the $\dot{\psi}$ which represent the drift velocity. Thus, the control variate is expected to be more effective if we apply it to a higher temperature and/or less collisional plasma, and we will report in the presentation the results of parameter survey calculations for the improved control variate. In addition, the multi-dimensional control variate method and the energy scattering for the marker particles are also implemented and results will be presented.

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