Particle Based Fluid Simulation for 3D Scene with Implicit Surfaces

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1 Introduction

The shape modeling technique based on the implicit surface representation is used for generating three-dimensional surface models that interpolate given sets of scattered points. A number of techniques for generating interpolating surfaces have been proposed and significant progress has been made in the last decade [1].

In addition, the implicit surface representation is also useful for interactive modeling of simple shapes [2] and three-dimensional scenes can be constructed as a set of implicit surfaces. For appropriate visualization of the three-dimensional scenes, some additional effects such as water flows need to be added depending on the situation. The promising method for generating water flow is the Smoothed Particle Hydrodynamics (SPH [3]): a particle-based fluid simulation method. In the standard SPH, the fluid is discretized into a set of particles and obstacles in the simulation field are assumed to be represented as sets of dense particles. Harada et al. proposed another approach for SPH simulation that enables simulation in polygon-based scenes [4]. In this approach, the particle force acted by polygon obstacles is incorporated in addition to the forces acted by the surrounding particles.

In this paper, we present a method of fluid simulation in three-dimensional scene with obstacles represented as implicit surfaces. In this method, the particle motion is formulated by adding the forces acted by the obstacles defined as implicit surfaces into the standard particle forces as in the SPH.

2 Three-Dimensional Scenes with Implicit Surfaces

We assume that all obstacles in a simulation field are defined as implicit surfaces, i.e., each surface is defined as

$$f(\boldsymbol{x}) = 0, \quad \boldsymbol{x} \in \mathbb{R}^3.$$

The sign of $f(\boldsymbol{x})$ is assumed to be positive inside the surface and negative outside. The quantities to be obtained from the obstacles in the SPH are the following two: one is the nearest distance between a particle and the obstacles, and the other is the direction from the particle to the nearest point on the obstacles. In our formulation, the nearest distance is estimated using Taubin distance [5], a linear approximation of the distance from a point $\boldsymbol{x} \in \mathbb{R}^3$ to its nearest surface point on an implicit surface. The estimated distance is defined as

$$d^{\mathrm{T}}(\boldsymbol{x}) = \frac{|f(\boldsymbol{x})|}{||\nabla f(\boldsymbol{x})||}.$$

Let $\boldsymbol{n}(\boldsymbol{x})$ be the unit vector directed from the nearest point on the obstacle to $\boldsymbol{x}$. Because the nearest point on the obstacles is not given, we use the gradient of $d^{\mathrm{T}}(\boldsymbol{x})$ at the particle point instead of the exact direction vector.

3 Fluid Simulation with Implicit Surfaces

The particle-based fluid simulation is formulated based on the SPH with effects of implicit surface obstacles. In this section, we describe the formulation of the particle motion with effects from obstacles using the quantities introduced in Sec. 2.

3.1 Governing Equation and Particle Motion

In the SPH, the formulation of the fluid simulation using SPH is based on the Navier-Stokes equation,

$$\rho \frac{D\boldsymbol{v}}{Dt} = -\nabla p + \mu \nabla^2 \boldsymbol{v} + \boldsymbol{g},$$

where ρ, $\boldsymbol{v}$, p, μ and $\boldsymbol{g}$ are density, velocity, pressure, viscosity coefficient and the gravity, respectively. By letting the positions of the particles at time t be $\{\boldsymbol{x}_i^t\}$, the corresponding discrete equation with a time interval Δt is obtained as

$$\begin{aligned} \boldsymbol{v}_i^{t+\Delta t} &= \boldsymbol{v}_i^t + \frac{\Delta t}{\rho_i}(\boldsymbol{f}_i^{\mathrm{pres}} + \boldsymbol{f}_i^{\mathrm{visc}} + \boldsymbol{f}_i^{\mathrm{grav}}), \\ \boldsymbol{x}_i^{t+\Delta t} &= \boldsymbol{x}_i^t + \boldsymbol{v}_i^{t+\Delta t}\Delta t, \end{aligned}$$

where $\boldsymbol{f}_i^{\mathrm{pres}}$, $\boldsymbol{f}_i^{\mathrm{visc}}$ and $\boldsymbol{f}_i^{\mathrm{grav}}$ are pressure, viscosity and gravity forces acted on the i-th particle, respectively, which are determined by the location of the particles at time t as described in the following sections. The simulation is performed by applying the formula repeatedly starting from a user given initial particle locations.

3.2 Formulation of Pressure Term

In this subsection, we formulate the approximately obtained pressure $\boldsymbol{f}_i^{\mathrm{pres}}$. Our formulation is based on the formula given in [3], i.e., the pressure is determined as the sum of two terms:

$$\boldsymbol{f}_i^{\mathrm{pres}} = \begin{cases} \hat{\boldsymbol{f}}_i^{\mathrm{pres}} + m_i \dfrac{(d^{\mathrm{T}}(\boldsymbol{x}_i) - r)}{\Delta t^2} \nabla d^{\mathrm{T}}(\boldsymbol{x}_i), & r > d^{\mathrm{T}}(\boldsymbol{x}_i), \\ \hat{\boldsymbol{f}}_i^{\mathrm{pres}}, & r \le d^{\mathrm{T}}(\boldsymbol{x}_i), \end{cases}$$

where $\hat{\boldsymbol{f}}_i^{\mathrm{pres}}$ is the pressure affected by other particles in the vicinity of the i-th (see [2] for detail) and r is a constant that determines the area under the influence of the obstacles.

3.3 Formulation of Viscous Term

The viscous term is also divided into two elements:

$$\boldsymbol{f}_i^{\mathrm{visc}} = \hat{\boldsymbol{f}}_i^{\mathrm{visc}} - \mu \boldsymbol{v}_i Z^{\mathrm{visc}}(d^{\mathrm{T}}(\boldsymbol{x}_i)),$$

where $\hat{\boldsymbol{f}}_i^{\mathrm{visc}}$ is the viscous force affected by its surrounding particles and $Z^{\mathrm{visc}}(d)$ is the function defined in [4] that determines the viscosity affected from the obstacles. Although the operand d is exactly obtained if the obstacles are polygon models, the exact distance is not obtained in the case of implicit surfaces. In our approach, we use the Taubin distance as the operand instead of the exact distance.

4 Results

We applied the particle-based simulation described in the previous section to the three-dimensional scene with an implicit surface defined as

$$f(\boldsymbol{x}) = \left\{1 - \left(\frac{x}{6}\right)^2 - \left(\frac{y}{3.5}\right)^2\right\} \left\{(x-3.9)^2 + y^2 - 1.44\right\}$$
$$\cdot \left(x^2 + y^2 - 1.44\right) \left\{(x+3.9)^2 + y^2 - 1.44\right\} - (20z)^2.$$

The initial locations of the SPH particles are set inside the surface with zero initial velocitys. The number of the particles is 28361. The results of the simulation are shown in Fig.1. From these results, we can see that the particles appropriately move at the vicinity of the surface and a natural motion can be obtained by our formulation.

5 Conclusion

In this paper, we presented the fluid simulation based on the SPH. We extended the particle-based simulation so that the simulation could be performed with obstacles expressed as implicit surfaces. The closest distance from a particle to the implicit surface is estimated using the Taubin distance and the direction of the force acted from the obstacles to a particle is estimated by the gradient of the Taubin distance at a particle point. Our numerical tests have shown that our algorithm works appropriately inside a surface defined as an implicit surface model.

References

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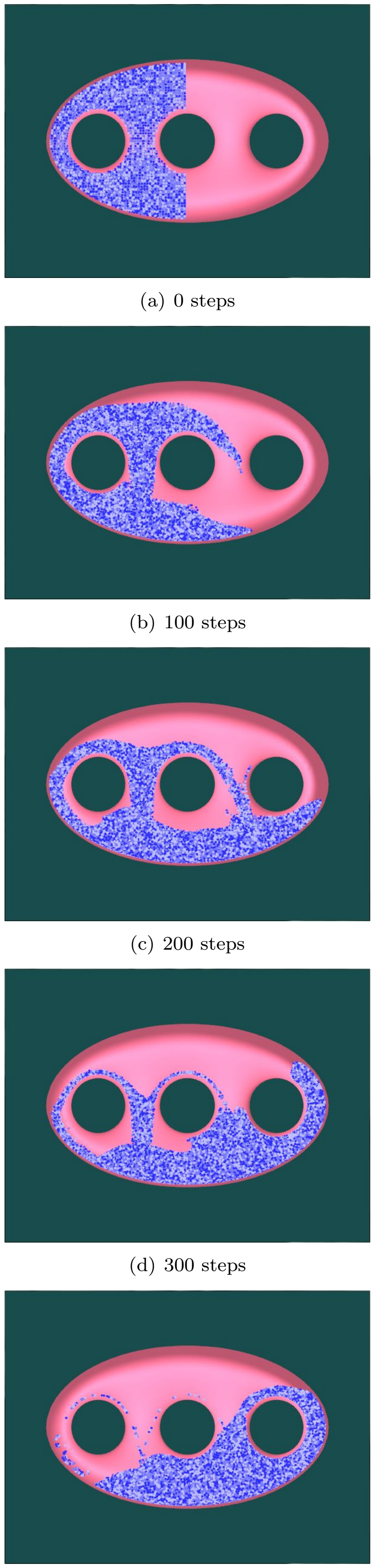

(a) 0 steps

(b) 100 steps

(c) 200 steps

(d) 300 steps

(e) 400 steps

Fig. 1. Result of a fluid simulation