

# Simulation of Vehicle Platoon Using Multi-vehicle Following Model

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## 1 Introduction

Traffic congestion in the urban city is becoming more terrible due to the increase of the vehicles. For overcoming this difficulty, Intelligent Transportation Systems (ITS) has been developing. Vehicle navigation system and electric toll collection system, which are well-known results of the ITS, are applied widely. One of the next ITS development is vehicle platoons. Vehicle Platoons decrease the distances between vehicles using electronic, and possibly mechanical, coupling. The use of the vehicle platoon can increase the traffic amount.

In this study, we will describe the vehicle velocity control of the vehicle platoon. The control algorithm is defined by the help of the optimal velocity (OV) model. This model controls the acceleration rate according to the difference between the vehicle velocity and the optimal velocity[1, 2]. In this study, the OV model is extended to the multi-leader model in which the velocity depends on the distance from multi-leader vehicles. The stability analysis of the model gives the stability region of the parameters. Finally, traffic simulation shows the effectiveness of the multi-leader vehicle following model to the vehicle platoon.

## 2 Velocity Control Algorithm

### 2.1 Optimal Velocity Model

The position of the vehicle  $n$  is given as  $x_n(t)$ . In Optimal velocity (OV) model[1], the acceleration rate  $\ddot{x}_n(t)$  is given as follows.

$$\ddot{x}_n(t) = a \{V(\Delta x_n) - \dot{x}_n(t)\} \quad (1)$$

$$\Delta x_n = x_{n-1}(t) - x_n(t) \quad (2)$$

where the notation  $\Delta x_n$ ,  $a$  and  $\ddot{x}_n$  denote the vehicle head distance, the driver's sensitivity and the acceleration rate, respectively. The optimal velocity function  $V(\Delta x_n)$  is given as

$$V(\Delta x_n) = \frac{V_{\max}}{2} [\tanh(\Delta x_n - x_c) + \tanh(x_c)] \quad (3)$$

where the parameter  $x_c$  denotes the safety distance.

### 2.2 OV Model of Vehicle Platoon

In the vehicle platoon, the lead vehicle follows the simple OV model. In the other vehicles, which are named as follower vehicles, the velocity depends on all leader vehicles.

The acceleration rate of the lead vehicle is given as follows.

$$\ddot{x}_1(t) = a_L [V_L(\Delta x_1) - \dot{x}_1(t)] \quad (4)$$

$$\Delta x_1 = x_0(t) - x_1(t) \quad (5)$$

$$V_L(\Delta x) = \frac{V_{\max}}{2} [\tanh(\Delta x - 2x_c) + \tanh(2x_c)] \quad (6)$$

where the variable  $x_1$  and  $x_0$  denote the positions of the lead vehicle and its leader vehicle, respectively. The function  $V_L$  is the optimal velocity function.

The acceleration rate of the follower vehicle is given as follows.

$$\ddot{x}_n(t) = \sum_{j=1}^k a_j [V_j(\Delta x_j) - \dot{x}_n(t)] \quad (7)$$

$$\Delta x_j = x_{n-j}(t) - x_n(t) \quad (8)$$

$$V_j(\Delta x) = \frac{V_{\max}}{2} [\tanh(\Delta x - jx_c) + \tanh(x_c)] \quad (9)$$

where the variable  $k$  is the total number of the leader vehicles. The function  $V_j$  is the optimal velocity function depending the distance  $\Delta x$  between the vehicle  $n$  and the lead vehicle  $n - j$ .

### 2.3 Stability Analysis

The sensitivity for the near leader vehicle is stronger than that for the far leader vehicle. Therefore, the following inequality can be considered.

$$a_5 \leq a_4 \leq a_3 \leq a_2 \leq a_1 \quad (10)$$

In equation (7), the steady state at  $x = x_0$  is defined so that all vehicles move at the same velocity with the identical head distance  $b = L/N$ . The variables  $L$  and  $N$  denote the road length and the total number of vehicles.

The perturbation  $y_n$  around the steady state  $x = x_0$  is considered as follows

$$x_n = x_0 + y_n \quad |y_n| \ll 1. \quad (11)$$

Substitution of equation (11) into equation (7) leads to

$$\ddot{y}_n(t) = \sum_{j=1}^k a_j [f_j(y_{n-j}(t) - y_n(t)) - \dot{y}_n(t)] \quad (12)$$

Fourier series of  $y_n$  has the solution

$$y_k(n, t) = \exp(i\alpha_k n + zt) \quad (13)$$

$$\alpha_k = \frac{2\pi}{N} k \quad k = 0, 1, 2, \dots, N-1 \quad (14)$$

Table 1. Sensitivities

$k = 1$	$a_1 = 3$
$k = 2$	$a_1 = 2, a_2 = 1$
$k = 3$	$a_1 = 3/2, a_2 = 1, a_3 = 1/2$
$k = 4$	$a_1 = 6/5, a_2 = 9/10, a_3 = 3/5, a_4 = 3/10$

Substitution of equation  $z = u + iv$  into equation (12) leads to

$$z^2 + \sum_{j=1}^k a_j f_j (1 - \cos \alpha_k j) - i \sum_{j=1}^m a_j f_j \sin(\alpha_k j) + z \sum_{j=1}^m a_j = 0. \quad (15)$$

At  $u = 0$ , equation (15) is on the limit state. Therefore, we have

$$\frac{\sigma_f}{\sigma_a} = \frac{\sigma_c}{\sigma_s} \quad (16)$$

where

$$\begin{aligned} \sigma_c &= \sum_{j=1}^k a_j (1 - \cos(\alpha_k j)) \\ \sigma_s &= \sum_{j=1}^k a_j (\sin(\alpha_k j)) \\ \sigma_a &= \sum_{j=1}^k a_j \\ \sigma_f &= \sum_{j=1}^k f_j \end{aligned}$$

## 2.4 Stability Condition

The stability conditions at  $k = 1, 2, 3$  and  $4$  are obtained from equation (16) as follows.

$$\left. \begin{aligned} k = 1 & \quad \frac{V'(b)}{a_1} < \frac{1}{2} \\ k = 2 & \quad \frac{V'_1(b) + V'_2(b)}{a_1} < \frac{27}{32} \\ k = 3 & \quad \frac{V'_1(b) + V'_2(b) + V'_3(b)}{a_1} < \frac{6}{5} \\ k = 4 & \quad \frac{V'_1(b) + V'_2(b) + V'_3(b) + V'_4(b)}{a_1} < \frac{25}{16} \end{aligned} \right\} \quad (17)$$

## 3 Simulation Result

Traffic simulation of the vehicle platoon on the one-lane road of 1000m length is performed. Maximum velocities of the lead vehicle and the other ones are specified as 13.88m/s (50km/h) and 16.66m/s (60km/h), respectively. Vehicle velocity is decreased to 11.11m/s (40km/h) when it pass from 600m point to 700m point. The sensitivities are shown in Table 1.

The traffic amount is shown in Fig.1. The figure is plotted with the traffic amount as the vertical axis and the vehicle density as the horizontal axis, respectively. The labels denote the following platoons.

- Case 1: Two Vehicle Group  
Two vehicles are grouped into one platoon. Each vehicle follows the single leader OV model.

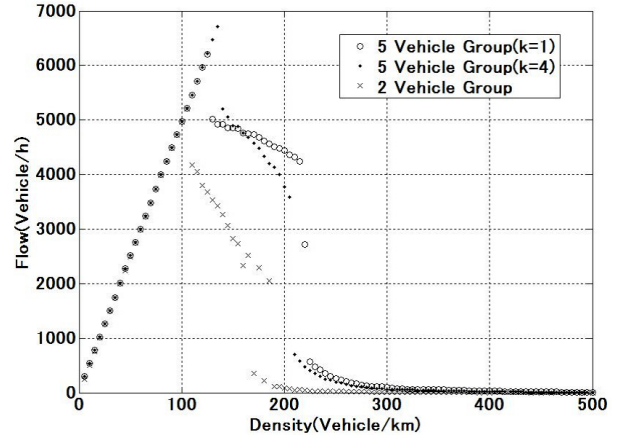


Fig. 1. Traffic flow

- Case 2: Five Vehicle Group ( $k = 1$ )  
Five vehicles are grouped into one platoon. Each vehicle follows the single leader OV model.
- Case 3: Five Vehicle Group ( $k = 4$ )  
Five vehicles are grouped into one platoon. Each vehicle follows the multi leader OV model.

Maximum traffic amounts of three cases are estimated as 5200 (Vehicle/h) in case 1, 6200 (Vehicle/h) in case 2, and 6700 (Vehicle/h) in case 3, respectively. The results show that the vehicle platoon and the multi-leader vehicle model are effective for increasing traffic amount.

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## 4 Conclusion

In this study, the velocity control in the vehicle platoons was described. The control algorithm was defined by the multi-leader optimal velocity model in which the velocity depends on the distance from multi-leader vehicles. The stability analysis of the model gives the stability condition of the model parameter.

In the simulation, three vehicle platoons were compared; two vehicles platoon according to single leader OV model, five vehicles platoon according to single leader OV model, and five vehicles platoon according to the multi-leader OV model. Maximum traffic amount of five vehicles platoon according to the multi-leader OV model was the largest among them. Therefore, we concluded that the vehicle platoon and the multi-leader vehicle model were effective for increasing traffic amount.

## References

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