Generalization of an edge coloring of graphs and channel assignment problem on multi-hop wireless networks

Hiroshi Tamura [1], Kazuyuki Miyakita [2], Keisuke Nakano [2], Masakazu Sengoku [2] and Shoji Shinoda [3]

[1] Faculty of Science and Engineering, Chuo University, Japan
[2] Niigata University, Japan, [3] Waseda University, Japan

1 Introduction

Multi-hop wireless networks [1] are autonomous systems of mobile nodes connected by wireless links and consist of nodes and wireless links. The nodes are classified into two kinds: one is a mobile terminal such as mobile phone, personal computer, handheld computer, Personal Digital Assistant (PDA) and so on, each of which has a router communication function for other terminals and the other is a special terminal that acts only as a router for mobile terminals and has a long-life or a permanent battery(Fig.1). Such multi-hop wireless networks are useful in various applications when cellular infrastructure is neither available nor realistic or when ubiquitous communication services are required without the presence or use of a fixed infrastructure.

For multi-hop wireless networks, we assign channels to communication between two terminals. This is a kind of edge coloring problem in graph theory. In the previous paper[2], we defined a new edge coloring problem called middle edge coloring and discussed the property of the coloring. In this paper, we generalize the coloring and consider the meaning of the generalization.

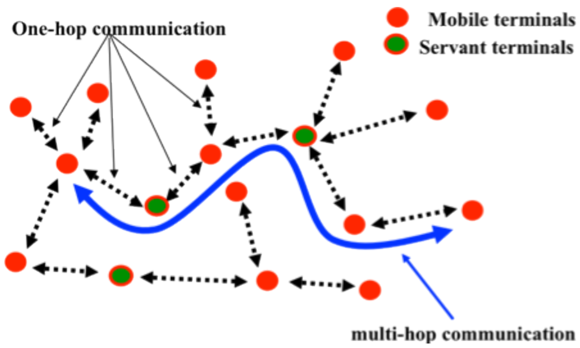

Fig.1 Multi-hop wireless network.

2 Preliminary

In this paper, we adopt undirected graphs as a model of multi-hop wireless networks. First, we define some terms of graph theory.

[Definition.1] Let G be an undirected graph. A conventional edge coloring of G is an assignment of colors to edges of G such that every two adjacent edges are assigned different colors.

[Definition.2] Let G be an undirected graph. A strong edge coloring of G is an assignment of colors to edges of G such that every two edges of distance at most two are assigned different colors. The distance of two edges is at most two means the edges are adjacent, or there is an edge between them.

The above two edge colorings are well-known. The following coloring are defined in [2].

[Definition.3] Let G be an undirected graph. If an assignment of colors to edges of G satisfies the following conditions, the coloring is called a middle edge coloring.

1) The assignment of colors is a conventional edge coloring.
2) Let (y,z) be an edge of G and be assigned color B. If an edge (x,y) and (z,u) are assigned color A(not B), x and u is not incident to edges assigned color B

Clearly, any middle edge coloring is a conventional edge coloring, and any strong edge coloring is a middle edge coloring. So, we call this coloring “middle” edge coloring.

3 A generalization of the middle edge coloring

The middle edge coloring prohibits the following coloring.

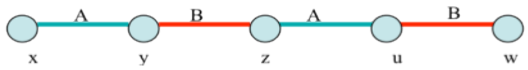

Fig.2 A coloring which is not middle edge coloring.

This is a coloring using two colors. So, we call this coloring two-alternate coloring.

So, a natural generalization of n-alternate coloring is as follows

[Definition.4] Let P_{2n} be a path whose length is 2n. The assigned colors to edges of P_{2n} are $A_1,A_2,\ldots,A_n$, $A_1,A_2,\ldots,A_n$ sequentially. This coloring is called n-alternate coloring.

Using the definition above, we define a new coloring.

[Definition.5] For a natural number n, an edge coloring that not include t-alternate coloring for any t ($\leq$n) is called n-seg coloring.

Clearly, any 1-seg coloring is a conventional coloring. So, n-seg coloring is a generalization of the conventional edge coloring.

n-alternate coloring and n-seg coloring are simple expansion. However, there is the following interesting property.

[Property 1] Let P_{2n} be a path whose vertices $v_0,v_1,\ldots,v_{2n}$ and edges are $e_1,e_2,\ldots,e_{2n}$ sequentially and assigned the distance $d(e_i)$ of edge e_i for any e_i. Moreover, P_{2n} is assigned n-alternate coloring. For $1 \leq i \leq n$, Let edge (v_{i-1},v_i) be assigned color A and (v_j,v_{j+1}) be also assigned color A. We define

ratio(v_i)=$d(v_i.v_j)/d(v_{i-1},v_i)$,

where, $d(v_i,v_j)$ is the distance between vertex v_i and v_j.

For $n \leq i \leq 2n-1$, Let edge (v_i,v_{i+1}) be assigned color A and (v_{j-1},v_j) be also assigned color A. We define

ratio(v_i)=$d(v_i.v_j)/d(v_i,v_{i+1})$.

Then,

minratio(P_{2n})={ratio(v) | v is in P_{2n}} is not greater than n-1.

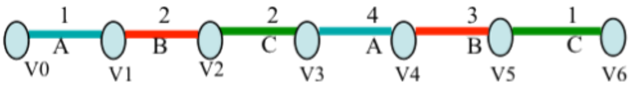

Fig.3 3-altanate coloring for P_6.

We explain the property using Fig.3. In Fig.3, a number on each edge represents the distanse of the edge and a letter under each edge represents the assigned color to the edge.

The edges (v_0,v_1) and (v_3,v_4) are assigned color A. And

$d(v_0,v_1)=1$, $d(v_1,v_3)=2+2=4$

Therefore ratio(v_1)=4/1=4. Other values are the following.

ratio(v_2)=(2+4)/2=3
ratio(v_3)=(4+3)/2=3.5
ratio(v_3)=(2+2)/4=1
ratio(v_4)=(2+4)/3=2
ratio(v_5)=(4+3)/1=7

About the middle vertex v_3 of the path, we calculate ratio(v_3) twice. In this case, minratio(P_6) is the following.

minratio(P_6)=min{4, 3, 3.5, 1, 2, 7}=1<n-1(=2).

In case of same distance for each edge, minratio(P_{2n})=n-1. So, the value n-1 is critical.

4 Generalization of n-seg coloring

In the previous section, we define n-seg coloring. However, the definition prohibits coloring limited very much. Here, we generalize the prohibited coloring.

[Definition.6] For graph G. n-Gseg coloring of G is an edge coloring satisfying the following condition.

For any path P_{2t} whose length is 2t ($1 \leq t \leq n$), on G, the number of colors on P_{2t} is greater than or equal to t+1.

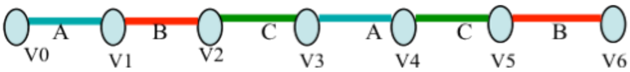

Fig.4 A coloring to Path P_6.

We explain the coloring. In Fig.4, the coloring of P_6 is a 3-seg coloring. Because, there is not 2-altanate coloring and 3-altanate coloring in P_6. However, the coloring is not 3-Gseg coloring. Because the number of colors on P_6 is three. Conversely, n-Gseg coloring is obviously n-seg coloring.

In case of n=1, the number of colors on P_2 is 2. Therefore, conventional edge coloring is equivalent to 1-Gseg coloring.

In case of n=2, the number of colors on P_2 is 2, and the number of colors on P_4 is 3 or 4. Therefore, middle edge coloring is equivalent to 2-Gseg coloring.

Every strong edge coloring is 2-Gseg coloring. However every 2-Gseg coloring is not always strong edge coloring.

In the previous paper, we show if a coloring is not middle edge coloring then the coloring is not applied to channel assignment of multi-hop wireless networks. However, in case of Gseg coloring, the property above does not hold. For example, we consider the following case.

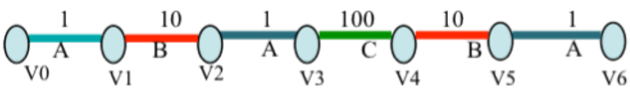

Fig.5 A coloring which is not Gseg coloring.

This coloring is not 3-Gseg coloring. However, this coloring can be applied to channel assignment of multi-hop wireless network because the distance (v_3,v_4) is very large.

In Fig.4, color C is assigned only to (v_3,v_4). This fact is very important for consideration of channel assignment. Because the color assigned only to one edge has nothing to do with the cochannel interference. Therefore we ignore the edge and the coloring to the edge in consideration with cochannel interferences. Therefore, we ignore the edge and the coloring of the edge when we consider the cochannel interference. As a result,

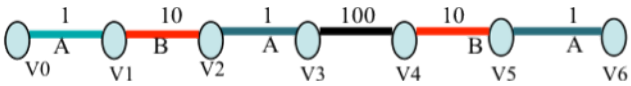

Fig.6 The coloring that is changed.

the coloring is as Fig.6. This coloring uses two colors and this is 2-Gseg coloring.

When we apply edge coloring to channel assignment, the following step is necessary.

1) We assign a color to every edge.
2) We ignore the color that is used only once.
3) We consider whether coloring is appropriate.

Using this model, we suppose an algorithm and evaluate it by computer simulation.

References

[1] C. E. Perkins, Ad Hoc Networking, Addison-Wesley, 2001.

[2] Hiroshi Tamura, Masakazu Sengoku and Shoji Shinoda, On an Edge Coloring between Conventional Edge Coloring and Strong Edge Coloring for Wireless Communications, Proc. 2011 Int. Tech. Conf. Circuits/Systems, Computers and Comm. (ITC-CSCC2011), pp.57-60, 2011.